Sampling and Estimation

Cambridge International AS & A Level - Mathematics 9709
Paper 6: Probability & Statistics 2 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 6 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Explain briefly why randomness is important when choosing a sample from a population. [3]

2 The random variable X has distribution $N(50, 8^2)$. A random sample of 16 observations is taken. Find the distribution of the sample mean $\overline{X}$, and find $P(\overline{X} > 52)$. [6]

3 A random sample of size 36 from a normal population with known standard deviation 6 has sample mean 72.4. Find a 95 percent confidence interval for the population mean. [5]

4 For a sample of 8 observations, $\sum x = 100$ and $\sum x^2 = 1280$. Find unbiased estimates of the population mean and variance. [6]

5 In a random sample of 200 voters, 124 support a proposal. Find an approximate 95 percent confidence interval for the population proportion supporting the proposal. [6]

6 A population has mean 12 and variance 16. A random sample of size 25 is taken. Use the Central Limit Theorem to estimate the probability that the sum of the sample observations is greater than 310. [6]

7 A random sample consists of the values $4, 7, 9, 10, 12$. Calculate unbiased estimates of the population mean and variance. [6]

8 A large random sample of size 64 has sample mean 15.2 and sample standard deviation 4.8. Find an approximate 95 percent confidence interval for the population mean. [5]

9 A population has known standard deviation 10. Find the minimum sample size needed for a 95 percent confidence interval for the mean to have margin of error less than 2. [5]

10 A website asks visitors to complete a voluntary survey about their weekly exercise. Explain why this sampling method may be unsatisfactory. [3]

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Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 Randomness helps ensure that each member of the population has a fair chance of being selected.
This reduces systematic bias in the sample.
It makes it more reasonable to use the sample to estimate features of the population.

2 $\overline{X} \sim N\left(50, \frac{8^2}{16}\right) = N(50, 4)$.

The standard deviation of $\overline{X}$ is 2.

$P(\overline{X} > 52) = P\left(Z > \frac{52 - 50}{2}\right)$.

$= P(Z > 1) = 0.159$ to 3 significant figures.

3 A 95 percent confidence interval is $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$.

$72.4 \pm 1.96\frac{6}{\sqrt{36}} = 72.4 \pm 1.96$.

The interval is $(70.44, 74.36)$.

4 The unbiased estimate of the population mean is $\bar{x} = \frac{100}{8} = 12.5$.

The unbiased estimate of the population variance is $s^2 = \frac{\sum x^2 - (\sum x)^2/n}{n - 1}$.

$s^2 = \frac{1280 - 100^2/8}{7} = \frac{30}{7}$.

$s^2 = 4.29$ to 3 significant figures.

5 $\hat{p} = \frac{124}{200} = 0.62$.

An approximate 95 percent confidence interval is $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$.

$= 0.62 \pm 1.96\sqrt{\frac{0.62(0.38)}{200}}$.

The interval is $(0.553, 0.687)$ to 3 significant figures.

6 Let S be the sum of the 25 observations.
By the Central Limit Theorem, S is approximately $N(25(12), 25(16)) = N(300, 400)$.

$P(S > 310) = P\left(Z > \frac{310 - 300}{20}\right)$.

$= P(Z > 0.5) = 0.309$ to 3 significant figures.

7 $\bar{x} = \frac{4 + 7 + 9 + 10 + 12}{5} = 8.4$.

The squared deviations from 8.4 sum to 37.2.

The unbiased estimate of the variance is $\frac{37.2}{5 - 1} = 9.3$.

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8 For a large sample, use $\bar{x} \pm 1.96\frac{s}{\sqrt{n}}$.

$15.2 \pm 1.96\frac{4.8}{8} = 15.2 \pm 1.176$.

The interval is $(14.024, 16.376)$, or approximately $(14.0, 16.4)$.

9 The margin of error is $1.96\frac{10}{\sqrt{n}}$.

Require $1.96\frac{10}{\sqrt{n}} < 2$.

$\sqrt{n} > 9.8$, so $n > 96.04$.

The minimum integer sample size is 97.

10 The sample is voluntary, so people with strong opinions or unusual exercise habits may be more likely to respond.
Only website visitors can be included, so some members of the population have no chance of selection.
The sample may therefore be biased and not representative of the target population.

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