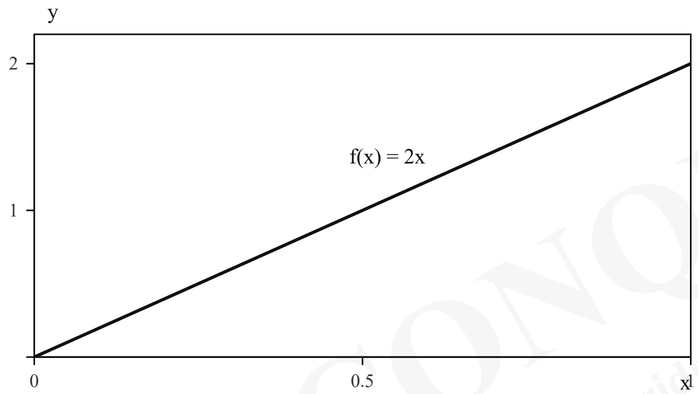
# Continuous Random Variables

Cambridge International AS & A Level - Mathematics 9709
Paper 6: Probability & Statistics 2 - Exam-style question bank

*Original Steps to Conquer practice questions in the style of recent 9709 Paper 6 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.*

## Questions

**1** The probability density function of $X$ is $f(x) = 2x$ for $0 < x < 1$, and $0$ otherwise. Find $P(0.2 < X < 0.8)$, $E(X)$, $\text{Var}(X)$ and the median of $X$. [8]




*Density function for Question 1.*

**2** The probability density function of $X$ is $f(x) = kx(4-x)$ for $0 < x < 4$, and $0$ otherwise. Find $k$, $P(1 < X < 3)$ and $E(X)$. [8]

**3** The probability density function of $X$ is $f(x) = \dfrac{3}{(x+1)^4}$ for $x \geq 0$. Find $P(X > 2)$ and the median of $X$. [6]

**4** The density function of $X$ is $f(x) = c(1+x)$ for $0 < x < 2$. Find $c$, $P(X > 1)$ and $E(X)$. [7]

**5** The density function of $X$ is $f(x) = \dfrac{3}{8}x^2$ for $0 < x < 2$. Find the median, $E(X)$ and $\text{Var}(X)$. [8]

**6** A continuous random variable $X$ has density $f(x) = \dfrac{1}{4}$ for $0 < x < 1$, $f(x) = \dfrac{3}{8}$ for $1 \leq x < 3$, and $0$ otherwise. Find $P(X < 2)$ and $E(X)$. [6]

**7** The density function of $X$ is $f(x) = c(3-x)$ for $0 < x < 3$. Find $c$ and the median of $X$. [7]

**8** The density function of $X$ is $f(x) = \dfrac{1}{2\sqrt{x}}$ for $0 < x < 1$. Find $P(X < 0.25)$ and $E(X)$. [5]

**9** The density function of $X$ is $f(x) = \frac{3}{4}(1 - x^2)$ for $-1 < x < 1$. Find $E(X)$ and $\mathrm{Var}(X)$. [6]

**10** The lifetime $T$ of a component has density $f(t) = 0.25e^{-0.25t}$ for $t \geq 0$. Find $P(T > 4)$ and $P(3 < T < 8)$. [6]

## Worked solutions

*Full methods with final answers. Other valid methods may also earn full marks.*

**1** $P(0.2 < X < 0.8) = \int_{0.2}^{0.8} 2x\,dx = [x^2]_{0.2}^{0.8} = 0.60$ .

$E(X) = \int_0^1 x(2x)\,dx = \int_0^1 2x^2\,dx = \frac{2}{3}$ .

$E(X^2) = \int_0^1 x^2(2x)\,dx = \int_0^1 2x^3\,dx = \frac{1}{2}$ .

$\text{Var}(X) = \frac{1}{2} - (\frac{2}{3})^2 = \frac{1}{18}$ .

For the median $m$, $\int_0^m 2x\,dx = \frac{1}{2}$ , so $m^2 = \frac{1}{2}$ .

$m = \dfrac{1}{\sqrt{2}}$ .

**2** $1 = \int_0^4 kx(4-x)\,dx = k\left[2x^2 - \dfrac{x^3}{3}\right]_0^4$ .

$1 = k \cdot \dfrac{32}{3}$ , so $k = \dfrac{3}{32}$ .

$P(1 < X < 3) = \dfrac{3}{32}\left[2x^2 - \dfrac{x^3}{3}\right]_1^3 = \dfrac{11}{16}$ .

$E(X) = \int_0^4 x \cdot \dfrac{3}{32}x(4-x)\,dx$ .

$= \dfrac{3}{32}\int_0^4 (4x^2 - x^3)\,dx = 2$ .

**3** $P(X > 2) = \int_2^\infty \dfrac{3}{(x+1)^4}\,dx$ .

$= \left[-(x+1)^{-3}\right]_2^\infty = \dfrac{1}{27}$ .

For the median $m$, $P(X > m) = \frac{1}{2}$ .

$(m+1)^{-3} = \frac{1}{2}$ .

$m = \sqrt[3]{2} - 1$ .

**4** $1 = \int_0^2 c(1+x)\,dx = c\left[x + \dfrac{x^2}{2}\right]_0^2 = 4c$ .

So $c = \frac{1}{4}$ .

$P(X > 1) = \int_1^2 \frac{1}{4}(1+x)\,dx = \frac{5}{8}$ .

$E(X) = \int_0^2 x \cdot \frac{1}{4}(1+x)\,dx$ .

$= \frac{1}{4}\left[\dfrac{x^2}{2} + \dfrac{x^3}{3}\right]_0^2 = \dfrac{7}{6}$ .

**5** For the median $m$, $\int_0^m \dfrac{3}{8}x^2\,dx = \frac{1}{2}$ .

$\dfrac{m^3}{8} = \frac{1}{2}$ , so $m = \sqrt[3]{4}$ .

$E(X) = \int_0^2 x \cdot \frac{3}{8}x^2\,dx = \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2 = \frac{3}{2}$ .

$E(X^2) = \int_0^2 x^2 \cdot \frac{3}{8}x^2\,dx = \frac{3}{8}\left[\frac{x^5}{5}\right]_0^2 = \frac{12}{5}$ .

$\text{Var}(X) = \frac{12}{5} - (\frac{3}{2})^2 = \frac{3}{20}$ .

**6** $P(X < 2) = \int_0^1 \frac{1}{4}\,dx + \int_1^2 \frac{3}{8}\,dx$ .

$P(X < 2) = \frac{1}{4} + \frac{3}{8} = \frac{5}{8}$ .

$E(X) = \int_0^1 \frac{x}{4}\,dx + \int_1^3 \frac{3x}{8}\,dx$ .

$= \frac{1}{8} + \frac{3}{16}(9-1) = \frac{13}{8}$ .

**7** $1 = \int_0^3 c(3-x)\,dx = c\left[3x - \frac{x^2}{2}\right]_0^3$ .

$1 = \frac{9}{2}c$ , so $c = \frac{2}{9}$ .

For the median $m$, $\int_0^m \frac{2}{9}(3-x)\,dx = \frac{1}{2}$ .

$\frac{2}{9}\left(3m - \frac{m^2}{2}\right) = \frac{1}{2}$ .

$2m^2 - 12m + 9 = 0$ .

The root in $(0,3)$ is $m = 3 - \frac{3\sqrt{2}}{2}$ .

**8** $P(X < 0.25) = \int_0^{0.25} \frac{1}{2\sqrt{x}}\,dx = [\sqrt{x}]_0^{0.25} = 0.5$ .

$E(X) = \int_0^1 x \cdot \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int_0^1 x^{1/2}\,dx$ .

$E(X) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$ .

**9** The density is symmetric about 0, so $E(X) = 0$ .

$E(X^2) = \int_{-1}^1 x^2 \cdot \frac{3}{4}(1-x^2)\,dx$ .

$= \frac{3}{4}\left(\int_{-1}^1 x^2\,dx - \int_{-1}^1 x^4\,dx\right)$ .

$= \frac{3}{4}\left(\frac{2}{3} - \frac{2}{5}\right) = \frac{1}{5}$ .

Thus $\text{Var}(X) = \frac{1}{5} - 0^2 = \frac{1}{5}$ .

**10** $P(T > 4) = \int_4^\infty 0.25e^{-0.25t}\,dt = e^{-1} = 0.368$ .

$P(3 < T < 8) = \int_3^8 0.25e^{-0.25t}\,dt$ .

$= [-e^{-0.25t}]_3^8 = e^{-0.75} - e^{-2} = 0.337$ to 3 significant figures.