The Poisson Distribution

Cambridge International AS & A Level - Mathematics 9709
Paper 6: Probability & Statistics 2 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 6 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

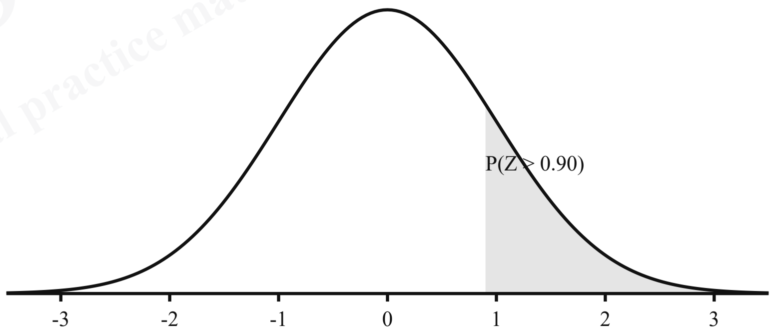
1 The random variable X has distribution $\mathrm{Po}(3.2)$. Find $P(X = 2)$, $P(X \le 1)$, $E(X)$ and $\mathrm{Var}(X)$. [6]

2 Calls arrive at a help desk at an average rate of 4.5 per hour. Assuming a Poisson model, find the probability that in a 20-minute period there are exactly 2 calls, and the probability that there is at least 1 call. [6]

3 Flaws occur in a length of fabric at an average rate of 0.8 flaws per metre. Find the probability that a randomly chosen 5 metre length contains no more than 2 flaws. [5]

4 A factory produces 200 components. Each component is defective with probability 0.015, independently of the others. Use a Poisson approximation to estimate the probability that at most 2 components are defective. [6]

5 Let $X \sim \mathrm{Po}(25)$. Use a normal approximation with continuity correction to estimate $P(X \ge 30)$. [6]



Normal approximation sketch for Question 5.

6 A Poisson random variable X satisfies $P(X = 0) = 0.1353$, correct to 4 decimal places. Find the mean of X and hence find $P(X = 3)$. [5]

7 Accidents occur on a particular road at an average rate of 2 per day. Find the probability that there are exactly 5 accidents in 3 days, and the probability that there are no accidents in a 12-hour period. [6]

8 A student suggests modelling the number of typing errors on a page by a Poisson distribution.

State two assumptions about the errors that would support this model. [3]

9 Let $X \sim \mathrm{Po}(40)$. Use a normal approximation with continuity correction to estimate $P(35 \leq X \leq 50)$. [7]

10 For $X \sim B(80, 0.04)$, explain why a Poisson approximation is reasonable, and use it to estimate $P(X = 4)$. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $P(X=2)=e^{-3.2}\dfrac{3.2^2}{2!}=0.209$ to 3 significant figures.

$P(X\le 1)=P(X=0)+P(X=1)=e^{-3.2}(1+3.2)=0.171$.

For $X\sim \mathrm{Po}(3.2)$, $E(X)=3.2$ and $\mathrm{Var}(X)=3.2$.

2 For 20 minutes, the mean number of calls is $4.5\times\frac{1}{3}=1.5$.

Let $X\sim \mathrm{Po}(1.5)$.

$P(X=2)=e^{-1.5}\dfrac{1.5^2}{2!}=0.251$.

$P(X\ge 1)=1-P(X=0)=1-e^{-1.5}=0.777$.

3 For 5 metres, the mean number of flaws is $0.8\times 5=4$.

Let $X\sim \mathrm{Po}(4)$.

$P(X\le 2)=e^{-4}\left(1+4+\dfrac{4^2}{2!}\right)$.

$P(X\le 2)=0.238$ to 3 significant figures.

4 Here $n=200$ is large and $p=0.015$ is small.

Use $X\sim \mathrm{Po}(np)=\mathrm{Po}(3)$.

$P(X\le 2)=e^{-3}\left(1+3+\dfrac{3^2}{2!}\right)$.

$P(X\le 2)=0.423$ to 3 significant figures.

5 Since $\lambda=25$ is large, use $Y\sim N(25,25)$.

$P(X\ge 30)\approx P(Y>29.5)$.

$z=\dfrac{29.5-25}{5}=0.90$.

Thus $P(X\ge 30)\approx P(Z>0.90)=0.184$.

6 For $X\sim \mathrm{Po}(\lambda)$, $P(X=0)=e^{-\lambda}$.

$e^{-\lambda}=0.1353$, giving $\lambda\approx 2.00$.

$P(X=3)=e^{-2}\dfrac{2^3}{3!}=0.180$ to 3 significant figures.

7 For 3 days, the mean is 6.

$P(X=5)=e^{-6}\dfrac{6^5}{5!}=0.161$.

For 12 hours, the mean is 1.

$P(X=0)=e^{-1}=0.368$.

8 Errors should occur independently of each other.

Errors should occur at a constant average rate over the page.

The chance of two or more errors in a very small part of the page should be negligible.

9 Since $\lambda = 40$ is large, use $Y \sim N(40, 40)$.

$P(35 \leq X \leq 50) \approx P(34.5 < Y < 50.5)$.

The corresponding z values are $\dfrac{34.5 - 40}{\sqrt{40}} = -0.870$ and $\dfrac{50.5 - 40}{\sqrt{40}} = 1.660$.

The probability is $\Phi(1.660) - \Phi(-0.870) = 0.759$ to 3 significant figures.

10 $n = 80$ is large and $p = 0.04$ is small, with $np = 3.2 < 5$.

Use $X \approx \mathrm{Po}(3.2)$.

$P(X = 4) \approx e^{-3.2}\dfrac{3.2^4}{4!}$.

$P(X = 4) = 0.178$ to 3 significant figures.