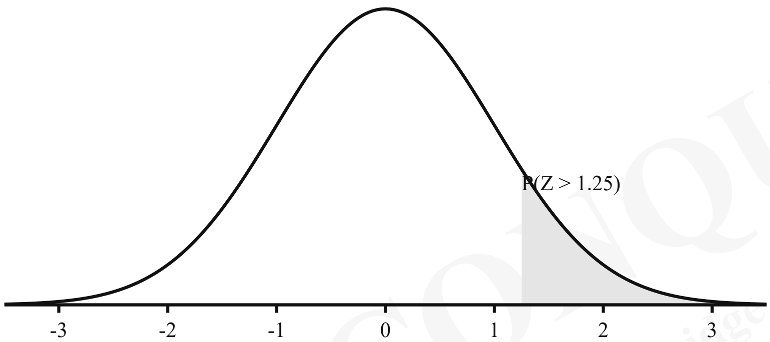
The Normal Distribution

Cambridge International AS & A Level - Mathematics 9709
Paper 5: Probability & Statistics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 5 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 The random variable X has distribution $N(50, 8^2)$. Find $P(X > 60)$ and $P(42 < X < 58)$. [6]



Standard normal sketch for the first probability.

2 If $X \sim N(50, 8^2)$, find the value of k such that $P(X < k) = 0.90$. [5]

3 The random variable X has distribution $N(\mu, 5^2)$. Given that $P(X < 38) = 0.2119$, find μ. [5]

4 The marks in a test are modelled by $X \sim N(100, 12^2)$. Find $P(90 < X < 115)$. [5]

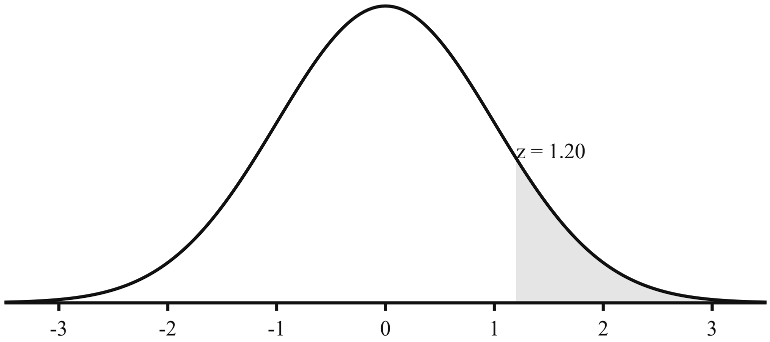
5 A binomial random variable $X \sim B(80, 0.35)$ is to be approximated by a normal distribution. State why the approximation is suitable, and estimate $P(X \geq 35)$ using a continuity correction. [7]

6 Let $X \sim B(200, 0.04)$. Use a normal approximation with continuity correction to estimate $P(X \leq 5)$. [6]

7 The weight W of a packet is normally distributed with mean 500 g and standard deviation 15 g. Find the probability that a packet weighs less than 475 g, and the probability that it weighs more than 530 g. [6]

8 The random variable X has distribution $N(100, \sigma^2)$. Given that $P(X > 118) = 0.0359$, find σ. [5]

9 Use a standard normal table to find $P(Z > 1.20)$. Show this probability on a sketch of the standard normal curve. [4]



Required shaded region for Question 9.

10 The lengths of metal rods are normally distributed with mean 12.4 cm and standard deviation 0.18 cm. Find the interval symmetric about the mean that contains approximately 95 percent of the lengths. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 For $X > 60$, standardise: $z = \dfrac{60 - 50}{8} = 1.25$.

$P(X > 60) = P(Z > 1.25) = 1 - \Phi(1.25) = 0.1056$.

For $42 < X < 58$, the corresponding z values are -1 and 1.

$P(42 < X < 58) = \Phi(1) - \Phi(-1) = 0.6827$.

2 $P(Z < z) = 0.90$ gives $z = 1.282$ from normal tables.

$\dfrac{k - 50}{8} = 1.282$.

$k = 50 + 8(1.282) = 60.3$ to 3 significant figures.

3 $P(Z < -0.8) = 0.2119$.

So $\dfrac{38 - \mu}{5} = -0.8$.

$38 - \mu = -4$.

$\mu = 42$.

4 The standardised values are $\dfrac{90 - 100}{12} = -0.833$ and $\dfrac{115 - 100}{12} = 1.25$.

$P(90 < X < 115) = \Phi(1.25) - \Phi(-0.833)$.

$= 0.692$ to 3 significant figures.

5 $np = 80(0.35) = 28$ and $nq = 80(0.65) = 52$, both greater than 5.

Use $Y \sim N(28, 18.2)$, since $npq = 80(0.35)(0.65) = 18.2$.

$P(X \geq 35) \approx P(Y > 34.5)$.

$z = \dfrac{34.5 - 28}{\sqrt{18.2}} = 1.524$.

Therefore $P(X \geq 35) \approx P(Z > 1.524) = 0.0638$.

6 $np = 8$ and $nq = 192$, so the normal approximation is suitable.

Use $Y \sim N(8, 7.68)$.

$P(X \leq 5) \approx P(Y < 5.5)$.

$z = \dfrac{5.5 - 8}{\sqrt{7.68}} = -0.902$.

Thus $P(X \leq 5) \approx \Phi(-0.902) = 0.183$.

7 For 475 g, $z = \dfrac{475 - 500}{15} = -1.667$.

$P(W < 475) = \Phi(-1.667) = 0.0478$.

For 530 g, $z = \dfrac{530 - 500}{15} = 2.00$.

$P(W > 530) = 1 - Phi(2.00) = 0.0228$.

8 $P(X > 118) = 0.0359$ means $P(Z < z) = 0.9641$.

From tables, $z = 1.80$.

$\frac{118 - 100}{\sigma} = 1.80$.

$\sigma = 10$.

9 From normal tables, $\Phi(1.20) = 0.8849$.

$P(Z > 1.20) = 1 - 0.8849 = 0.1151$.

The sketch should shade the right-hand tail beyond $z = 1.20$.

10 For the central 95 percent of a normal distribution, use $z = 1.96$.

The interval is $12.4 \pm 1.96(0.18)$.

$1.96(0.18) = 0.3528$.

The interval is from 12.0472 to 12.7528 cm.

To 3 significant figures, this is 12.0 cm to 12.8 cm.