Discrete Random Variables

Cambridge International AS & A Level - Mathematics 9709
Paper 5: Probability & Statistics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 5 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 The probability distribution of X is shown below. Find k, $E(X)$ and $\text{Var}(X)$. [7]

x	0	1	2	3
P(X=x)	k	2k	3k	4k

2 A game has possible gains X dollars of -1, 2 and 5 with probabilities 0.5, 0.3 and 0.2 respectively. Find $E(X)$ and $\text{Var}(X)$. [6]

3 Let $X \sim B(8, 0.3)$. Find $P(X=2)$, $P(X \leq 1)$, $E(X)$ and $\text{Var}(X)$. [7]

4 If $X \sim B(20, p)$ and $E(X) = 6$, find p and $\text{Var}(X)$. [4]

5 A geometric random variable X has success probability 0.25. Find $P(X=5)$, $P(X>3)$ and $E(X)$. [6]

6 The distribution of X is given below. Find k, $E(X)$ and $\text{Var}(X)$. [7]

x	1	2	3	4
P(X=x)	0.20	k	0.35	0.15

7 Three fair coins are tossed. Let X be the number of heads obtained. Draw up the probability distribution of X, and find $E(X)$. [6]

8 Each item produced by a factory is defective with probability 0.08, independently of all other items. A sample of 12 items is taken. Find the probability that at least one item is defective. [4]

9 A geometric random variable has expectation 5. Find its success probability p and find $P(X \leq 3)$. [5]

10 Let $X \sim B(10, 0.4)$. Find $P(3 \leq X \leq 5)$. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $k + 2k + 3k + 4k = 1$, so $10k = 1$ and $k = 0.1$.

$E(X) = 0(k) + 1(2k) + 2(3k) + 3(4k) = 20k = 2$.

$E(X^2) = 1^2(2k) + 2^2(3k) + 3^2(4k) = 50k = 5$.

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 5 - 2^2 = 1$.

2 $E(X) = (-1)(0.5) + 2(0.3) + 5(0.2) = 1.1$.

$E(X^2) = 1(0.5) + 4(0.3) + 25(0.2) = 6.7$.

$\mathrm{Var}(X) = 6.7 - (1.1)^2 = 5.49$.

3 $P(X = 2) = \binom{8}{2}(0.3)^2(0.7)^6 = 0.296$ to 3 significant figures.

$P(X \le 1) = P(X = 0) + P(X = 1)$.

$= (0.7)^8 + \binom{8}{1}(0.3)(0.7)^7 = 0.255$ to 3 significant figures.

$E(X) = np = 8(0.3) = 2.4$.

$\mathrm{Var}(X) = npq = 8(0.3)(0.7) = 1.68$.

4 $E(X) = np = 20p = 6$, so $p = 0.3$.

$\mathrm{Var}(X) = npq = 20(0.3)(0.7) = 4.2$.

5 For $X \sim \mathrm{Geo}(0.25)$, $P(X = r) = (0.75)^{r-1}(0.25)$.

$P(X = 5) = (0.75)^4(0.25) = 0.0791$ to 3 significant figures.

$P(X > 3)$ means the first 3 trials are failures, so $P(X > 3) = (0.75)^3 = 0.422$.

$E(X) = \dfrac{1}{p} = 4$.

6 $0.20 + k + 0.35 + 0.15 = 1$, so $k = 0.30$.

$E(X) = 1(0.20) + 2(0.30) + 3(0.35) + 4(0.15) = 2.45$.

$E(X^2) = 1(0.20) + 4(0.30) + 9(0.35) + 16(0.15) = 6.95$.

$\mathrm{Var}(X) = 6.95 - (2.45)^2 = 0.9475$.

7 The possible values of X are $0, 1, 2, 3$.

The probabilities are $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$.

This is the distribution $B(3, 0.5)$.

$E(X) = np = 3(0.5) = 1.5$.

8 Let X be the number of defective items. Then $X \sim B(12, 0.08)$.

$P(X \ge 1) = 1 - P(X = 0)$.

$= 1 - (0.92)^{12} = 0.632$ to 3 significant figures.

9 For a geometric random variable, $E(X) = \dfrac{1}{p}$.

$\dfrac{1}{p} = 5$, so $p = 0.2$.

$P(X \le 3) = 1 - P(X > 3)$.

$= 1 - (0.8)^3 = 0.488$.

10 $P(3 \le X \le 5) = P(X = 3) + P(X = 4) + P(X = 5)$.

$= \binom{10}{3}(0.4)^3(0.6)^7 + \binom{10}{4}(0.4)^4(0.6)^6 + \binom{10}{5}(0.4)^5(0.6)^5$.

$= 0.666$ to 3 significant figures.