Probability

Cambridge International AS & A Level - Mathematics 9709
Paper 5: Probability & Statistics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 5 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Two fair six-sided dice are thrown. Find the probability that the total score is greater than 9. [3]

2 A bag contains 5 red balls, 3 blue balls and 2 green balls. Two balls are chosen at random without replacement. Find the probability that they are the same colour. [5]

3 Events A and B have $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. Find $P(A \cup B)$ and determine whether A and B are independent. [5]

4 A disease affects 8 percent of a population. A test gives a positive result for 95 percent of people with the disease and for 12 percent of people without the disease. Find the probability that a randomly chosen person tests positive, and the probability that a person has the disease given that the test is positive. [7]

5 Four cards are chosen at random from a standard pack of 52 cards. Find the probability that exactly two of the cards are aces. [5]

6 A biased spinner has probabilities $0.1, 0.2, 0.3, 0.4$ of landing on $1, 2, 3, 4$ respectively. The spinner is spun twice. Find the probability that the sum is 5 and the probability that both spins show the same number. [5]

7 Two fair dice are thrown. Given that at least one die shows a 6, find the probability that the total score is 8. [5]

8 A component is made by machine M_1 with probability 0.7 and by machine M_2 with probability 0.3. The probabilities of a defective component are 0.02 for M_1 and 0.05 for M_2. Find the probability that a component is defective, and the probability that a defective component was made by M_2. [6]

9 A team of 4 is chosen at random from 10 candidates, including Alice and Ben. Find the probability that neither Alice nor Ben is chosen, and the probability that exactly one of them is chosen. [6]

10 Events A and B are independent, $P(A) = 0.4$ and $P(A \cup B) = 0.72$. Find $P(B)$. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 Totals greater than 9 are 10, 11 and 12.

The numbers of outcomes are 3, 2 and 1 respectively.

Probability $= \dfrac{6}{36} = \dfrac{1}{6}$.

2 The total number of ways to choose 2 balls is $\binom{10}{2} = 45$.

Same colour choices $= \binom{5}{2} + \binom{3}{2} + \binom{2}{2} = 10 + 3 + 1 = 14$.

The required probability is $\dfrac{14}{45}$.

3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A \cup B) = 0.6 + 0.5 - 0.3 = 0.8$.

$P(A)P(B) = 0.6 \times 0.5 = 0.3$.

Since $P(A \cap B) = P(A)P(B)$, the events are independent.

4 $P(\text{positive}) = 0.08(0.95) + 0.92(0.12)$.

$P(\text{positive}) = 0.076 + 0.1104 = 0.1864$.

$P(\text{disease} \mid \text{positive}) = \dfrac{0.08(0.95)}{0.1864}$.

$= 0.408$ to 3 significant figures.

5 Choose 2 aces from 4 and 2 non-aces from 48.

The number of favourable hands is $\binom{4}{2}\binom{48}{2}$.

The total number of hands is $\binom{52}{4}$.

Probability $= \dfrac{\binom{4}{2}\binom{48}{2}}{\binom{52}{4}} = 0.0250$ to 3 significant figures.

6 For a sum of 5, the possible ordered pairs are $(1,4), (2,3), (3,2), (4,1)$.

Probability $= 0.1(0.4) + 0.2(0.3) + 0.3(0.2) + 0.4(0.1) = 0.20$.

For both spins the same, probability $= 0.1^2 + 0.2^2 + 0.3^2 + 0.4^2$.

$= 0.01 + 0.04 + 0.09 + 0.16 = 0.30$.

7 There are 11 equally likely outcomes in which at least one die shows 6.

These are the 6 outcomes with the first die 6 and the 5 additional outcomes with only the second die 6.

The outcomes with total 8 and at least one 6 are $(6,2)$ and $(2,6)$.

Required probability $= \dfrac{2}{11}$.

8 $P(\text{defective}) = 0.7(0.02) + 0.3(0.05) = 0.029$.

$P(M_2 \mid \text{defective}) = \dfrac{0.3(0.05)}{0.029}$.

$= 0.517$ to 3 significant figures.

9 The total number of teams is $\binom{10}{4} = 210$.

Neither Alice nor Ben chosen: choose all 4 from the other 8, giving $\binom{8}{4} = 70$ teams.

Probability $= \frac{70}{210} = \frac{1}{3}$.

Exactly one of Alice and Ben: choose 1 of the 2, and 3 of the other 8.

Probability $= \frac{\binom{2}{1}\binom{8}{3}}{\binom{10}{4}} = \frac{112}{210} = \frac{8}{15}$.

10 Let $P(B) = p$.

Since A and B are independent, $P(A \cap B) = 0.4p$.

$P(A \cup B) = 0.4 + p - 0.4p = 0.72$.

$0.6p = 0.32$, so $p = \frac{8}{15}$.