Permutations and Combinations

Cambridge International AS & A Level - Mathematics 9709
Paper 5: Probability & Statistics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 5 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Seven students are to stand in a line. Find the number of possible arrangements. [2]

2 Find the number of distinct arrangements of the letters of the word NEEDLESS. [4]

3 Find the number of distinct arrangements of the letters of NEEDLESS in which no two Es are next to each other. [5]

4 A committee of 4 people is to be chosen from 6 boys and 5 girls. Find the number of committees containing at least 2 girls. [5]

5 How many 4-digit numbers greater than 5000 can be formed from the digits $0, 1, 2, 3, 4, 5, 6, 7$ if no digit is repeated? [5]

6 Five men and four women are to stand in a line. Find the number of arrangements in which all four women stand together. [4]

7 Ten people are to sit in two distinct rows of five seats. Two particular people must sit in the same row. Find the number of possible seating arrangements. [6]

8 A hand of 5 cards is selected from 7 red cards and 5 blue cards. Find the number of hands containing exactly 3 red cards. [3]

9 Six different books are to be arranged on a shelf. Two particular books must not be next to each other. Find the number of possible arrangements. [5]

10 A password consists of 3 different letters chosen from A, B, C, D, E and F, followed by 2 digits chosen from 0 to 9. Digits may be repeated. Find the number of possible passwords. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 The number of arrangements of 7 distinct students is $7!$.
$7! = 5040$.

2 The word NEEDLESS has 8 letters, with 3 Es and 2 Ss repeated.
The number of distinct arrangements is $\frac{8!}{3!2!} = 3360$.

3 Arrange the non-E letters N, D, L, S, S first.
These can be arranged in $\frac{5!}{2!} = 60$ ways.

There are 6 gaps around these letters in which the 3 Es can be placed.
Choose 3 of the 6 gaps: $\binom{6}{3} = 20$.
Total arrangements $= 60 \times 20 = 1200$.

4 The possible numbers of girls are 2, 3 or 4.
Number $= \binom{5}{2}\binom{6}{2} + \binom{5}{3}\binom{6}{1} + \binom{5}{4}$.
$= 10(15) + 10(6) + 5 = 215$.

5 The first digit must be 5, 6 or 7, giving 3 choices.
After the first digit is chosen, there are 7 choices, then 6 choices, then 5 choices.
The number of integers is $3 \times 7 \times 6 \times 5 = 630$.

6 Treat the four women as one block.
There are then 6 units to arrange: the block and the five men.
These can be arranged in $6!$ ways, and the women within the block in $4!$ ways.
Total arrangements $= 6!4! = 17280$.

7 The two particular people can occupy two seats in the first row or two seats in the second row.
The number of choices of their two seats is $\binom{5}{2} + \binom{5}{2} = 20$.
The two people can be arranged in those seats in $2!$ ways.
The remaining 8 people can be arranged in the remaining seats in $8!$ ways.
Total arrangements $= 20 \times 2 \times 8! = 1612800$.

8 Choose 3 red cards from 7 and 2 blue cards from 5.
Number of hands $= \binom{7}{3}\binom{5}{2} = 35 \times 10 = 350$.

9 Total arrangements without restriction are $6! = 720$.
If the two particular books are together, treat them as one block.
There are then 5 units to arrange, and the two books within the block can be swapped.
Together arrangements $= 5! \times 2 = 240$.
Required number $= 720 - 240 = 480$.

10 The letters can be arranged in ${}^6P_3 = 6 \times 5 \times 4 = 120$ ways.
There are $10^2 = 100$ choices for the two digits.

Total passwords $= 120 \times 100 = 12000$.