Kinematics of Motion in a Straight Line

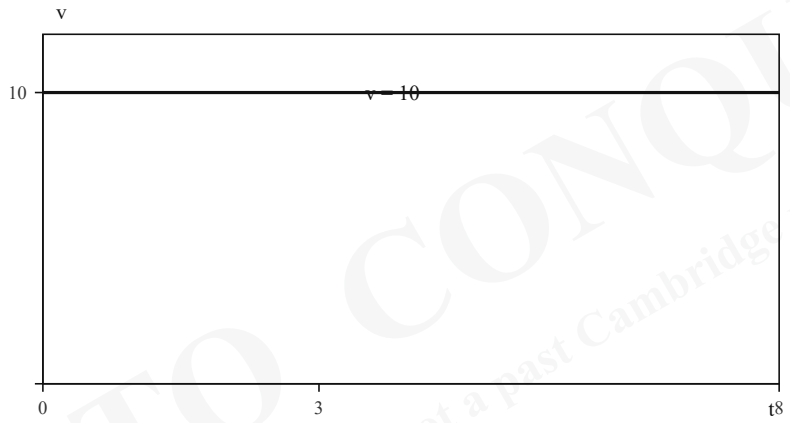
Cambridge International AS & A Level - Mathematics 9709
Paper 4: Mechanics - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 4 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 A particle moves in a straight line with initial velocity 5 m s^{-1} and constant acceleration 2 m s^{-2}. Find its velocity and displacement after 6 seconds. [4]

2 The velocity-time graph of a particle is shown below. Find the total distance travelled in the first 8 seconds. [5]



Velocity is 10 m s^-1 from t = 0 to t = 8.

3 The displacement s metres of a particle at time t seconds is given by $s = t^3 - 6t^2 + 9t$. Find the velocity and acceleration at time t, and find the times when the particle is instantaneously at rest. [6]

4 A particle has velocity $v = 12 - 3t$ for $0 \le t \le 5$. Find its displacement and total distance travelled in this interval. [6]

5 A car starts from rest and reaches a speed of 20 m s^{-1} in 8 seconds with constant acceleration. Find the acceleration and the distance travelled. [4]

6 Particle A starts from rest at a point O and moves with constant acceleration 2 m s^{-2}. At the same instant, particle B is 30 m ahead of O and moves in the same direction with constant speed 4 m s^{-1}. Find when A catches B. [6]

7 A ball is projected vertically upwards with speed 20 m s^{-1}. Taking $g = 10$ m s^{-2}, find the greatest height reached and the time until it returns to its point of projection. [5]

8 A particle moves with velocity $v = kt^2$. Given that $v = 18$ when $t = 3$, find k and the distance travelled in the first 3 seconds. [5]

9 A train travelling at $30\,\text{m s}^{-1}$ decelerates uniformly to rest in a distance of 150 m. Find its acceleration and the time taken to stop. [5]

10 A particle moves from rest with constant acceleration. It travels 72 m in 6 seconds. Find its acceleration and its speed after 6 seconds. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $v = u + at = 5 + 2(6) = 17$ m s^{-1}.

$s = ut + \frac{1}{2}at^2 = 5(6) + \frac{1}{2}(2)(6^2) = 66$ m.

2 The distance travelled is the area under the velocity-time graph.

The graph is a rectangle of width 8 and height 10.

Distance $= 8 \times 10 = 80$ m.

3 $v = \dfrac{ds}{dt} = 3t^2 - 12t + 9$.

$a = \dfrac{dv}{dt} = 6t - 12$.

At rest, $v = 0$, so $3t^2 - 12t + 9 = 0$.

$t^2 - 4t + 3 = 0$, giving $t = 1$ or $t = 3$.

4 Displacement $= \int_0^5 (12 - 3t)\,dt = [12t - \frac{3}{2}t^2]_0^5 = 22.5$ m.

The velocity is zero when $12 - 3t = 0$, so $t = 4$.

Distance from 0 to 4 is the area of a triangle: $\frac{1}{2}(4)(12) = 24$ m.

Distance from 4 to 5 is $\frac{1}{2}(1)(3) = 1.5$ m.

Total distance $= 25.5$ m.

5 $a = \dfrac{v - u}{t} = \dfrac{20}{8} = 2.5$ m s^{-2}.

$s = \dfrac{u + v}{2}t = \dfrac{20}{2}(8) = 80$ m.

6 For A, displacement from O is $s_A = \frac{1}{2}(2)t^2 = t^2$.

For B, displacement from O is $s_B = 30 + 4t$.

At the catch-up time, $t^2 = 30 + 4t$.

$t^2 - 4t - 30 = 0$.

$t = 2 + \sqrt{34} = 7.83$ s to 3 significant figures.

7 At the greatest height, $v = 0$.

$0 = 20^2 - 2(10)s$, so $s = 20$ m.

The time to reach the greatest height is $t = \dfrac{20}{10} = 2$ s.

The total time to return is 4 s.

8 $18 = k(3^2)$, so $k = 2$.

Distance $= \int_0^3 2t^2\,dt = \left[\dfrac{2t^3}{3}\right]_0^3$.

Distance $= 18$ m.

9 $v^2 = u^2 + 2as$ gives $0 = 30^2 + 2a(150)$.

$a = -3$ m s^{-2}.

$v = u + at$ gives $0 = 30 - 3t$.

$t = 10$ s.

10 $s = ut + \frac{1}{2}at^2$ and $u = 0$.

$72 = \frac{1}{2}a(36)$, so $a = 4$ m s^{-2}.

$v = u + at = 4(6) = 24$ m s^{-1}.