Forces and Equilibrium

Cambridge International AS & A Level - Mathematics 9709
Paper 4: Mechanics - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 4 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Two forces of magnitudes 6 N and 8 N act at right angles on a particle. Find the magnitude and direction of their resultant. [4]

2 A particle is in equilibrium under three forces. Two of the forces are 12 N due east and 5 N due north. Find the third force. [4]

3 A block of weight 50 N rests on a rough horizontal surface. A horizontal force of 20 N acts on the block and the block remains in equilibrium. Find the frictional force and the magnitude of the contact force exerted by the surface on the block. [5]

4 A particle of weight 30 N rests on a smooth plane inclined at 30° to the horizontal. It is held in equilibrium by a force P acting up the plane. Find P and the normal reaction. [5]

5 A particle of mass 5 kg is on a rough horizontal plane. The coefficient of friction is 0.4. Find the least horizontal force needed to put the particle in limiting equilibrium. [4]

6 A particle of mass 10 kg rests on a rough plane inclined at 20° to the horizontal. The coefficient of friction is 0.3. A force P acts up the plane and the particle is about to move up the plane. Find P. [6]

7 A particle of weight 40 N is held in equilibrium by a string making an angle of 30° above the horizontal and by a horizontal force P. Find the tension in the string and the value of P. [5]

8 A particle of mass 4 kg is in equilibrium on a rough horizontal surface under a horizontal force of 10 N. Find the magnitude of the contact force exerted by the surface and the angle it makes with the vertical. [6]

9 A particle rests on a rough plane inclined at 25° to the horizontal. Find the least possible coefficient of friction for equilibrium. [4]

10 Forces of 7 N, 9 N and F N act on a particle in equilibrium. The 7 N and 9 N forces act at right angles. Find F. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 Resultant magnitude $= \sqrt{6^2 + 8^2} = 10$ N.

If the 6 N force is horizontal, $\tan\theta = \frac{8}{6}$.

$\theta = 53.1^\circ$ above the direction of the 6 N force.

2 The resultant of the given two forces has components $(12, 5)$.

For equilibrium, the third force must have components $(-12, -5)$.

Its magnitude is $\sqrt{12^2 + 5^2} = 13$ N.

It acts in the opposite direction to the resultant, towards the south-west.

3 Horizontal equilibrium gives friction $= 20$ N, opposing the applied force.

Vertical equilibrium gives normal reaction $R = 50$ N.

The contact force is the resultant of friction and normal reaction.

Magnitude $= \sqrt{20^2 + 50^2} = 53.9$ N.

4 Resolving parallel to the plane, $P = 30\sin 30^\circ = 15$ N.

Resolving perpendicular to the plane, $R = 30\cos 30^\circ$.

$R = 26.0$ N to 3 significant figures.

5 The weight is $5g = 50$ N, so $R = 50$ N.

Limiting friction is $F = \mu R = 0.4(50) = 20$ N.

The least horizontal force is 20 N.

6 The normal reaction is $R = 100\cos 20^\circ$.

Since the particle is about to move up the plane, friction acts down the plane.

Limiting friction $= 0.3R = 30\cos 20^\circ$.

Resolving up the plane, $P = 100\sin 20^\circ + 30\cos 20^\circ$.

$P = 62.4$ N to 3 significant figures.

7 Let the tension be T.

Vertical equilibrium gives $T\sin 30^\circ = 40$, so $T = 80$ N.

Horizontal equilibrium gives $P = T\cos 30^\circ$.

$P = 69.3$ N to 3 significant figures.

8 The normal reaction is $R = 4g = 40$ N.

The frictional force is 10 N.

Contact force magnitude $= \sqrt{40^2 + 10^2} = 41.2$ N.

If the contact force makes angle θ with the vertical, $\tan\theta = \frac{10}{40}$.

$\theta = 14.0^\circ$.

9 For limiting equilibrium down the plane, $mg\sin 25^\circ = \mu mg\cos 25^\circ$.

$\mu = \tan 25^\circ$.

$\mu = 0.466$ to 3 significant figures.

10 The resultant of the 7 N and 9 N forces has magnitude $\sqrt{7^2 + 9^2}$.

F must be equal and opposite to this resultant.

$F = \sqrt{130} = 11.4$ N to 3 significant figures.