Integration

Cambridge International AS & A Level - Mathematics 9709
Paper 3: Pure Mathematics 3 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 3 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Find $\int \left(4e^{2x} - \frac{3}{2x+1} + 5\sec^2(5x)\right) dx$. [5]

2 Evaluate $\int_0^{\pi/4} \sin 2x \, dx$. [3]

3 Evaluate $\int_0^2 \frac{x}{x^2+4} \, dx$, giving your answer in exact form. [4]

4 Using partial fractions, find $\int \frac{7x+1}{(x+1)(2x-1)} \, dx$. [5]

5 Evaluate $\int_0^{\pi/2} \sin^2 x \, dx$. [4]

6 Find $\int xe^{2x} \, dx$. [5]

7 Evaluate $\int_1^e \ln x \, dx$. [4]

8 Using the substitution $u = \sin x$, evaluate $\int_0^{\pi/2} \sin^3 x \cos x \, dx$. [4]

9 Find the exact area enclosed by the curve $y = e^x$, the line $y = 1$, and the lines $x = 0$ and $x = \ln 3$. [5]

10 The region under the curve $y = 2e^{-x}$ from $x = 0$ to $x = \ln 2$ is rotated through 360° about the x-axis. Find the exact volume generated. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $\int 4e^{2x}\,dx = 2e^{2x}$.

$\int -\dfrac{3}{2x+1}\,dx = -\dfrac{3}{2}\ln|2x+1|$.

$\int 5\sec^2(5x)\,dx = \tan 5x$.

So the integral is $2e^{2x} - \dfrac{3}{2}\ln|2x+1| + \tan 5x + C$.

2 $\int \sin 2x\,dx = -\frac{1}{2}\cos 2x$.

$\int_0^{\pi/4} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/4}$.

$= -\frac{1}{2}\cos\frac{\pi}{2} + \frac{1}{2}\cos 0 = \frac{1}{2}$.

3 Recognise the integrand as $\dfrac{f'(x)}{2f(x)}$, where $f(x) = x^2 + 4$.

$\int \dfrac{x}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4)$.

$\left[\frac{1}{2}\ln(x^2+4)\right]_0^2 = \frac{1}{2}(\ln 8 - \ln 4)$.

The value is $\frac{1}{2}\ln 2$.

4 From partial fractions, $\dfrac{7x+1}{(x+1)(2x-1)} = \dfrac{2}{x+1} + \dfrac{3}{2x-1}$.

Therefore the integral is $2\ln|x+1| + \dfrac{3}{2}\ln|2x-1| + C$.

5 Use $\sin^2 x = \dfrac{1-\cos 2x}{2}$.

$\int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \dfrac{1-\cos 2x}{2}\,dx$.

$= \left[\dfrac{x}{2} - \dfrac{\sin 2x}{4}\right]_0^{\pi/2}$.

The value is $\dfrac{\pi}{4}$.

6 Use integration by parts with $u = x$ and $dv = e^{2x}dx$.

Then $du = dx$ and $v = \frac{1}{2}e^{2x}$.

$\int xe^{2x}\,dx = \dfrac{x}{2}e^{2x} - \int \dfrac{1}{2}e^{2x}\,dx$.

$= \dfrac{x}{2}e^{2x} - \dfrac{1}{4}e^{2x} + C$.

7 Using integration by parts, $\int \ln x\,dx = x\ln x - x + C$.

$\int_1^e \ln x\,dx = [x\ln x - x]_1^e$.

At $x = e$ this is $e - e = 0$; at $x = 1$ this is $0 - 1 = -1$.

The value is 1.

8 Let $u = \sin x$, so $du = \cos x\,dx$.

When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$.

The integral becomes $\int_0^1 u^3\,du$.

This is $\left[\frac{1}{4}u^4\right]_0^1 = \frac{1}{4}$.

9 For $0 \le x \le \ln 3$, $e^x \ge 1$.

Area $= \int_0^{\ln 3}(e^x - 1)\,dx$.

$= [e^x - x]_0^{\ln 3}$.

$= (3 - ln3) - (1 - 0) = 2 - ln3$.

10 Volume $= \pi\int_0^{\ln 2}(2e^{-x})^2\,dx$.

$= 4\pi\int_0^{\ln 2} e^{-2x}\,dx$.

$= 4\pi\left[-\frac{1}{2}e^{-2x}\right]_0^{\ln 2}$.

$= 2\pi(1 - e^{-2\ln 2}) = 2\pi(1 - \frac{1}{4}) = \dfrac{3\pi}{2}$.