Differentiation

Cambridge International AS & A Level - Mathematics 9709
Paper 3: Pure Mathematics 3 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 3 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Differentiate $y = x^2e^{-3x}$ with respect to x. [3]

2 The curve $y = x\ln x - 2x$, where $x > 0$, has one stationary point. Find its coordinates and determine its nature. [6]

3 For $x > 0$, the curve $y = \dfrac{\ln x}{x}$ has a stationary point. Find its exact coordinates. [5]

4 Differentiate $y = \sin 2x + \ln(3x - 1)$ with respect to x. [3]

5 A curve is defined parametrically by $x = t^2 + 1$, $y = t^3 - 3t$. Find the equation of the tangent to the curve at the point where $t = 2$. [6]

6 The curve $x^2 + y^2 = xy + 7$ passes through the point $(3, 2)$. Find the gradient of the normal to the curve at this point. [6]

7 For $y = e^x \sin x$, find $\dfrac{d^2y}{dx^2}$. [5]

8 The tangent to the curve $y = \ln x$ at the point where $x = a$ passes through the origin. Find a and the equation of the tangent. [6]

9 A curve is defined by $x = e^t$, $y = te^t$. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ in terms of t. [6]

10 Find the gradient of the curve $y = \tan^{-1}(2x)$ at the point where $x = \dfrac{1}{2}$. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 Use the product rule.

$\frac{dy}{dx} = 2xe^{-3x} + x^2(-3e^{-3x})$.

$\frac{dy}{dx} = e^{-3x}(2x - 3x^2)$.

2 $\frac{dy}{dx} = \ln x + 1 - 2 = \ln x - 1$.

At a stationary point, $\ln x = 1$, so $x = e$.

$y = e\ln e - 2e = e - 2e = -e$.

$\frac{d^2y}{dx^2} = \frac{1}{x}$, which is positive at $x = e$.

The stationary point is $(e, -e)$ and it is a minimum.

3 $y = (\ln x)x^{-1}$.

$\frac{dy}{dx} = \frac{1}{x}x^{-1} - (\ln x)x^{-2} = \frac{1 - lnx}{x^2}$.

Setting the derivative equal to zero gives $\ln x = 1$, so $x = e$.

$y = \frac{1}{e}$.

The stationary point is $(e, \frac{1}{e})$.

4 $\frac{d}{dx}(\sin 2x) = 2\cos 2x$.

$\frac{d}{dx}\ln(3x - 1) = \frac{3}{3x - 1}$.

So $\frac{dy}{dx} = 2\cos 2x + \frac{3}{3x - 1}$.

5 $\frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 3t^2 - 3$.

$\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$.

At $t = 2$, the gradient is $\frac{12 - 3}{4} = \frac{9}{4}$.

The point is $(5, 2)$.

The tangent is $y - 2 = \frac{9}{4}(x - 5)$.

6 Differentiate implicitly: $2x + 2y\frac{dy}{dx} = y + x\frac{dy}{dx}$.

$(2y - x)\frac{dy}{dx} = y - 2x$.

$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$.

At $(3, 2)$, $\frac{dy}{dx} = \frac{2-6}{4-3} = -4$.

The normal gradient is $\frac{1}{4}$.

7 $\frac{dy}{dx} = e^x \sin x + e^x \cos x = e^x(\sin x + cosx)$.

$\frac{d^2y}{dx^2} = e^x(\sin x + cosx) + e^x(\cos x - \sin x)$.

$\frac{d^2y}{dx^2} = 2e^x \cos x$.

8 The gradient at $x = a$ is $\frac{1}{a}$, and the point is $(a, \ln a)$.

The tangent is $y - \ln a = \frac{1}{a}(x - a)$.

Since it passes through $(0, 0)$, $-\ln a = -1$.

Thus $\ln a = 1$, so $a = e$.

The tangent is $y - 1 = \frac{1}{e}(x - e)$, or $y = \frac{x}{e}$.

9 $\frac{dx}{dt} = e^t$ and $\frac{dy}{dt} = e^t + te^t = e^t(t+1)$.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = t + 1$.

$\frac{d^2y}{dx^2} = \frac{d}{dt}(t+1) \div \frac{dx}{dt}$.

$\frac{d^2y}{dx^2} = \frac{1}{e^t} = e^{-t}$.

10 $\frac{d}{dx}\tan^{-1}(u) = \frac{u'}{1+u^2}$.

Here $u = 2x$, so $\frac{dy}{dx} = \frac{2}{1+4x^2}$.

At $x = \frac{1}{2}$, $\frac{dy}{dx} = \frac{2}{1+1} = 1$.