Trigonometry

Cambridge International AS & A Level - Mathematics 9709
Paper 3: Pure Mathematics 3 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 3 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Given that $\tan\theta = \dfrac{3}{4}$ and that $180^\circ < \theta < 270^\circ$, find the exact values of $\sin 2\theta$ and $\cos 2\theta$. [5]

2 Show that $\dfrac{1+\tan^2 x}{1+\cot^2 x} = \tan^2 x$. [3]

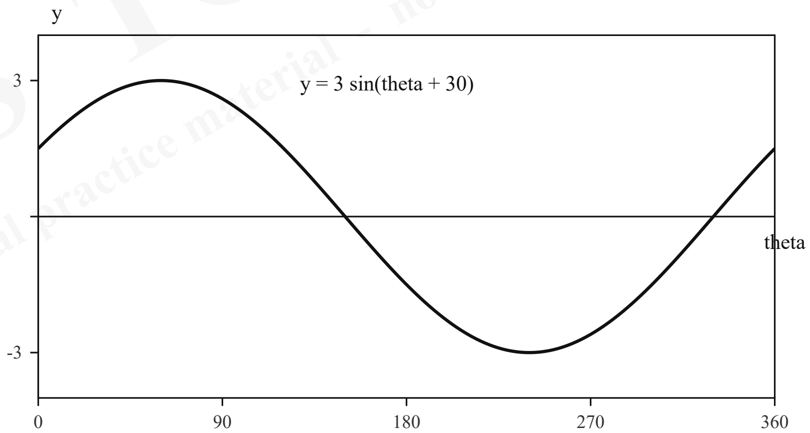
3 Solve $2\sec^2\theta - 5\tan\theta = 1$ for $0^\circ \le \theta < 360^\circ$. [6]

4 Use the addition formula for tangent to find the exact value of $\tan 15^\circ$. [4]

5 Solve $3\sin x + 4\cos x = 2$ for $0^\circ \le x < 360^\circ$. [6]

6 Solve $\sin 2x = \cos x$ for $0^\circ \le x < 360^\circ$. [5]

7 The graph of $y = 3\sin(\theta + 30^\circ)$, for $0^\circ \le \theta \le 360^\circ$, is shown below. State the amplitude and the values of θ for which the maximum value occurs in this interval. [4]



Graph for Question 7.

8 Solve $\tan\theta + \cot\theta = 4$ for $0^\circ < \theta < 180^\circ$. [5]

9 Solve $2\cos^2 x + sinx - 1 = 0$ for $0^\circ \le x < 360^\circ$. [5]

10 Angles A and B are acute, with $\tan A = \dfrac{1}{2}$ and $\tan B = 3$. Find the exact value of $\tan(A+B)$ and explain why $A+B$ is obtuse. [4]

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Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 In the third quadrant, $\sin\theta = -\frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$.

$\sin 2\theta = 2\sin\theta\cos\theta = 2(-\frac{3}{5})(-\frac{4}{5}) = \frac{24}{25}$.

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$.

2 Use $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \operatorname{cosec}^2 x$.

$$\frac{1+\tan^2 x}{1+\cot^2 x} = \frac{\sec^2 x}{\operatorname{cosec}^2 x} = \frac{1/\cos^2 x}{1/\sin^2 x}.$$

$$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x.$$

3 Using $\sec^2\theta = 1 + \tan^2\theta$ gives $2(1+\tan^2\theta) - 5\tan\theta = 1$.

Let $t = \tan\theta$. Then $2t^2 - 5t + 1 = 0$.

$$t = \frac{5 \pm \sqrt{17}}{4}.$$

For $t = \dfrac{5+\sqrt{17}}{4}$, $\theta = 66.3^\circ, 246.3^\circ$.

For $t = \dfrac{5-\sqrt{17}}{4}$, $\theta = 12.4^\circ, 192.4^\circ$.

4 $15^\circ = 45^\circ - 30^\circ$.

$$\tan 15^\circ = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}.$$

$$= \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}.$$

Rationalising gives $\tan 15^\circ = 2 - \sqrt{3}$.

5 Write $3\sin x + 4\cos x = 5\sin(x+\alpha)$, where $\cos\alpha = \frac{3}{5}$ and $\sin\alpha = \frac{4}{5}$.

Thus $\alpha = 53.1^\circ$.

$5\sin(x + 53.1^\circ) = 2$, so $\sin(x + 53.1^\circ) = 0.4$.

$x + 53.1^\circ = 23.6^\circ$ or 156.4° modulo 360°.

Hence $x = 330.4^\circ$ or 103.3°.

6 $\sin 2x = 2\sin x\cos x$.

So $2\sin x\cos x = \cos x$.

$\cos x(2\sin x - 1) = 0$.

Thus $\cos x = 0$ or $\sin x = \frac{1}{2}$.

$x = 90^\circ, 270^\circ, 30^\circ, 150^\circ$.

7 The amplitude is 3.

The maximum value occurs when $\sin(\theta + 30^\circ) = 1$.

$\theta + 30^\circ = 90^\circ + 360^\circ k$.

In the given interval, $\theta = 60^\circ$.

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8 Let $t = \tan\theta$. Then $t + \dfrac{1}{t} = 4$.

$t^2 - 4t + 1 = 0$, so $t = 2 \pm \sqrt{3}$.

$\tan 15^\circ = 2 - \sqrt{3}$ and $\tan 75^\circ = 2 + \sqrt{3}$.

Since both tangent values are positive in the first quadrant, $\theta = 15^\circ$ or 75°.

9 Use $\cos^2 x = 1 - \sin^2 x$.

$2(1 - \sin^2 x) + \sin x - 1 = 0$.

$2\sin^2 x - sinx - 1 = 0$.

$(2\sin x + 1)(\sin x - 1) = 0$.

$\sin x = 1$ or $\sin x = -\frac{1}{2}$.

$x = 90^\circ, 210^\circ, 330^\circ$.

10 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

$= \dfrac{\frac{1}{2} + 3}{1 - \frac{1}{2} \cdot 3} = \dfrac{7/2}{-1/2} = -7$.

Since $\tan(A+B) < 0$ and both A and B are acute, $A+B$ lies in the second quadrant.

Therefore $A+B$ is obtuse.

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