Logarithmic and Exponential Functions

Cambridge International AS & A Level - Mathematics 9709
Paper 3: Pure Mathematics 3 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 3 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

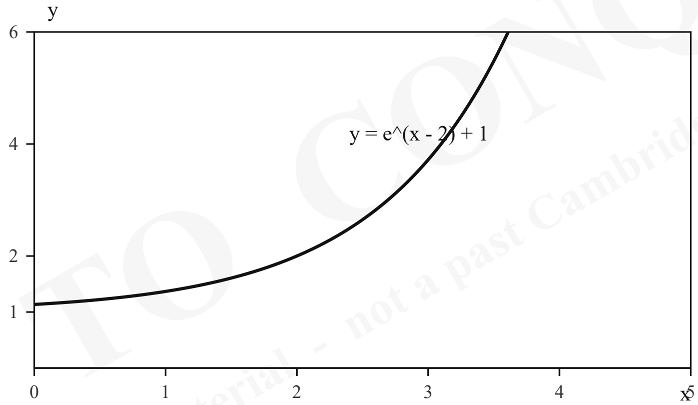
Questions

1 Solve the equation $\ln(2x-1)+\ln(x+3)=\ln 14$. [5]

2 Solve the equation $e^{2x}-5e^{x}+6=0$. [4]

3 Solve the inequality $3^{2x-1}<5$, giving your answer in exact logarithmic form. [4]

4 The curve $y=e^{x-2}+1$ is shown below. State its horizontal asymptote and find the exact value of x for which $y=4$. [4]



Graph of y = e^(x - 2) + 1.

5 A relationship between positive variables x and y has the form $y=kx^{n}$. The points $(2,9)$ and $(8,72)$ lie on its graph. Find the exact values of k and n. [6]

6 A curve has equation $y=ka^{x}$, where k and a are positive constants. The curve passes through $(1,6)$ and $(4,162)$. Find k and a. [5]

7 Solve the equation $\ln x=2\ln 3-\ln(x-4)$. [5]

8 A quantity N is modelled by $N=80e^{-0.35t}+20$, where t is measured in hours. Find the value of t when $N=35$. [4]

9 When $\ln y$ is plotted against x, the points lie on a straight line with gradient -0.4 and intercept 2.3 on the $\ln y$ axis. Write y in the form Ae^{Bx} and find the value of y when $x=5$. [5]

10 Find the exact solution of $4^{x+1}=8^{2x-1}$. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 The domain requires $x > \frac{1}{2}$.

$\ln(2x-1) + \ln(x+3) = \ln((2x-1)(x+3))$.

So $(2x-1)(x+3) = 14$.

$2x^2 + 5x - 3 = 14$, hence $2x^2 + 5x - 17 = 0$.

$x = \dfrac{-5 \pm \sqrt{161}}{4}$. Only the value greater than $\frac{1}{2}$ is valid.

$x = \dfrac{-5 + \sqrt{161}}{4}$.

2 Let $u = e^x$, where $u > 0$.

$u^2 - 5u + 6 = 0$, so $(u-2)(u-3) = 0$.

$e^x = 2$ or $e^x = 3$.

Therefore $x = \ln 2$ or $x = \ln 3$.

3 Since $3 > 1$, logarithms preserve the inequality.

$(2x-1)\ln 3 < \ln 5$.

$2x - 1 < \dfrac{\ln 5}{\ln 3}$.

Hence $x < \dfrac{1}{2}\left(1 + \dfrac{\ln 5}{\ln 3}\right)$.

4 As $x \to -\infty$, $e^{x-2} \to 0$, so the horizontal asymptote is $y = 1$.

For $y = 4$, $e^{x-2} + 1 = 4$.

$e^{x-2} = 3$, so $x - 2 = \ln 3$.

Hence $x = 2 + \ln 3$.

5 Taking logarithms gives $\ln y = \ln k + n \ln x$.

Using the two points, $\dfrac{72}{9} = \left(\dfrac{8}{2}\right)^n$.

$8 = 4^n$, so $n = \frac{3}{2}$.

Using $(2, 9)$, $9 = k2^{3/2}$.

$k = \dfrac{9}{2\sqrt{2}} = \dfrac{9\sqrt{2}}{4}$.

6 $6 = ka$ and $162 = ka^4$.

Dividing gives $27 = a^3$.

Thus $a = 3$.

Since $6 = 3k$, $k = 2$.

7 The domain requires $x > 4$.

$2\ln 3 = \ln 9$, so $\ln x + \ln(x-4) = \ln 9$.

Thus $x(x-4) = 9$.

$x^2 - 4x - 9 = 0$, giving $x = 2 \pm \sqrt{13}$.

Only $x = 2 + \sqrt{13}$ satisfies $x > 4$.

8 $80e^{-0.35t} + 20 = 35$.

$e^{-0.35t} = \frac{15}{80} = \frac{3}{16}$.

$-0.35t = \ln\left(\frac{3}{16}\right)$.

$t = \frac{\ln(16/3)}{0.35} = 4.78$ hours, to 3 significant figures.

9 $\ln y = 2.3 - 0.4x$.

Therefore $y = e^{2.3}e^{-0.4x}$, so $A = e^{2.3}$ and $B = -0.4$.

When $x = 5$, $y = e^{2.3-2} = e^{0.3}$.

$y = 1.35$ to 3 significant figures.

10 Write both sides as powers of 2.

$4^{x+1} = (2^2)^{x+1} = 2^{2x+2}$.

$8^{2x-1} = (2^3)^{2x-1} = 2^{6x-3}$.

Thus $2x + 2 = 6x - 3$, so $x = \frac{5}{4}$.