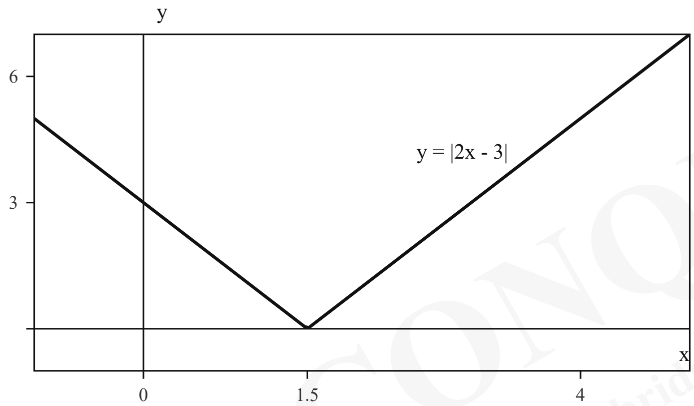
Algebra

Cambridge International AS & A Level - Mathematics 9709
Paper 3: Pure Mathematics 3 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 3 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 The function f is defined by $f(x) = |2x - 3|$. (a) Sketch the graph of $y = f(x)$, stating the coordinates of its vertex. [2] (b) Solve the inequality $|2x - 3| < x + 1$. [4]



Graph used for Question 1(a).

2 Solve the equation $|3x - 2| = |x + 4|$. [4]

3 The polynomial $P(x) = 2x^4 - 3x^3 + ax^2 + bx + 6$ leaves remainders 10 and 16 when divided by $x - 2$ and $x + 1$ respectively. Find a and b, and hence find the remainder when $P(x)$ is divided by $x^2 - x - 2$. [7]

4 The polynomial $x^3 + px^2 + qx - 12$ has a factor $x - 2$. When it is divided by $x + 1$ the remainder is -36. Find p and q, and factorise the polynomial completely. [7]

5 Express $\dfrac{7x+1}{(x+1)(2x-1)}$ in partial fractions. [4]

6 Find constants A, B and C such that

$$\frac{3x^2+8x+7}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}.$$

[5]

7 Decompose $\dfrac{3x^2+2x+5}{(x-1)(x^2+4)}$ into partial fractions. [5]

8 Use the binomial expansion to expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3. State the set of values of x for which the expansion is valid. [5]

9 Expand $\frac{1}{(2+x)^2}$ in ascending powers of x up to and including the term in x^3. State the range of values of x for which the expansion is valid. [6]

10 Use the expansion of $(1+3x)^{\frac{1}{2}}$ up to the term in x^3 to find an approximation to $\sqrt{1.06}$. Give your answer to 6 decimal places. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 (a) The graph is a V-shape with vertex where $2x-3=0$, so the vertex is $(\frac{3}{2},0)$.

(b) If $x<\frac{3}{2}$, then $|2x-3|=3-2x$. Hence $3-2x<x+1$, giving $x>\frac{2}{3}$.

If $x\geq\frac{3}{2}$, then $|2x-3|=2x-3$. Hence $2x-3<x+1$, giving $x<4$.

Combining the intervals gives $\frac{2}{3}<x<4$.

2 Squaring both sides gives $(3x-2)^2=(x+4)^2$.

Thus $3x-2=x+4$ or $3x-2=-(x+4)$.

The two solutions are $x=3$ and $x=-\frac{1}{2}$.

3 By the remainder theorem, $P(2)=10$ and $P(-1)=16$.

$P(2)=14+4a+2b=10$, so $2a+b=-2$.

$P(-1)=11+a-b=16$, so $a-b=5$.

Solving gives $a=1$ and $b=-4$.

Since $x^2-x-2=(x-2)(x+1)$, the remainder has form $Rx+S$.

$2R+S=10$ and $-R+S=16$, giving $R=-2$ and $S=14$.

The remainder is $-2x+14$.

4 Let $f(x)=x^3+px^2+qx-12$.

$f(2)=0$ gives $8+4p+2q-12=0$, so $2p+q=2$.

$f(-1)=-36$ gives $-1+p-q-12=-36$, so $p-q=-23$.

Solving gives $p=-7$ and $q=16$.

$f(x)=x^3-7x^2+16x-12=(x-2)(x^2-5x+6)$.

Hence $f(x)=(x-2)^2(x-3)$.

5 Let $\dfrac{7x+1}{(x+1)(2x-1)}=\dfrac{A}{x+1}+\dfrac{B}{2x-1}$.

Then $7x+1=A(2x-1)+B(x+1)$.

Using $x=-1$ gives $-6=-3A$, so $A=2$.

Using $x=\frac{1}{2}$ gives $\frac{9}{2}=\frac{3}{2}B$, so $B=3$.

Therefore $\dfrac{7x+1}{(x+1)(2x-1)}=\dfrac{2}{x+1}+\dfrac{3}{2x-1}$.

6 $3x^2+8x+7=A(x+2)^2+B(x+1)(x+2)+C(x+1)$.

Putting $x=-1$ gives $2=A$, so $A=2$.

Putting $x=-2$ gives $3=-C$, so $C=-3$.

Comparing coefficients of x^2 gives $3=A+B$, so $B=1$.

Thus $A=2$, $B=1$, $C=-3$.

7 Let $\dfrac{3x^2+2x+5}{(x-1)(x^2+4)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+4}$.

$3x^2+2x+5=A(x^2+4)+(Bx+C)(x-1)$.

Putting $x = 1$ gives $10 = 5A$, so $A = 2$.

Expanding gives $3x^2 + 2x + 5 = 2x^2 + 8 + Bx^2 + (C - B)x - C$.

Comparing coefficients gives $B = 1$ and $C = 3$.

So the partial fractions are $\frac{2}{x-1} + \frac{x+3}{x^2+4}$.

8 $(1-u)^{-\frac{1}{2}} = 1 + \frac{1}{2}u + \frac{3}{8}u^2 + \frac{5}{16}u^3 + \cdots$.

Putting $u = 2x$ gives $(1-2x)^{-\frac{1}{2}} = 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \cdots$.

The expansion is valid when $|2x| < 1$, so $|x| < \frac{1}{2}$.

9 $\frac{1}{(2+x)^2} = \frac{1}{4}(1 + \frac{x}{2})^{-2}$.

$(1+u)^{-2} = 1 - 2u + 3u^2 - 4u^3 + \cdots$.

Therefore $\frac{1}{(2+x)^2} = \frac{1}{4}\left(1 - x + \frac{3}{4}x^2 - \frac{1}{2}x^3 + \cdots\right)$.

Hence $\frac{1}{(2+x)^2} = \frac{1}{4} - \frac{1}{4}x + \frac{3}{16}x^2 - \frac{1}{8}x^3 + \cdots$.

The expansion is valid when $|\frac{x}{2}| < 1$, so $|x| < 2$.

10 $(1+3x)^{\frac{1}{2}} = 1 + \frac{1}{2}(3x) - \frac{1}{8}(3x)^2 + \frac{1}{16}(3x)^3 + \cdots$.

So $(1+3x)^{\frac{1}{2}} = 1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3 + \cdots$.

For $\sqrt{1.06}$, take $3x = 0.06$, so $x = 0.02$.

$\sqrt{1.06} \approx 1 + 0.03 - 0.00045 + 0.0000135 = 1.0295635$.

To 6 decimal places, $\sqrt{1.06} \approx 1.029564$.