Integration

Cambridge International AS & A Level - Mathematics 9709
Paper 2: Pure Mathematics 2 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 2 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Find $\int \left(6e^{3x} + \frac{4}{2x-1} - 5\sin(2x+1)\right)dx$. [5]

2 Evaluate $\int_0^{\pi/4} \cos 2x \, dx$. [3]

3 Find $\int \sec^2(4x-1)\, dx$. [3]

4 Use a trigonometric identity to evaluate $\int_0^{\pi/2} \cos^2 x, dx$. [4]

5 Evaluate $\int_1^3 \frac{5}{2x+1}\, dx$, giving your answer in exact form. [4]

6 Use the trapezium rule with 4 equal intervals to estimate $\int_0^2 e^{-x}\, dx$. Give your answer to 3 decimal places. [6]

x	**0**	**0.5**	**1.0**	**1.5**	**2.0**
e^{-x}	1.000	0.607	0.368	0.223	0.135

7 Use the trapezium rule with 3 equal intervals to estimate $\int_1^4 \ln x, dx$. [6]

x	1	2	3	4
ln x	0	0.693	1.099	1.386

8 Find the area bounded by the curve $y = e^{x}$, the x-axis, and the lines $x = 0$ and $x = \ln 2$. [4]

9 Find $\int (3\sin x - 4\cos 3x), dx$. [4]

10 The graph of $y = \ln x$ is concave down for $x > 0$. Explain whether the trapezium rule gives an over-estimate or an under-estimate for $\int_{1}^{4} \ln x, dx$. [2]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $\int 6e^{3x}dx = 2e^{3x}$.

$\int \frac{4}{2x-1}dx = 2\ln|2x-1|$.

$\int -5\sin(2x+1)dx = \frac{5}{2}\cos(2x+1)$.

The integral is $2e^{3x} + 2\ln|2x-1| + \frac{5}{2}\cos(2x+1) + C$.

2 $\int \cos 2x, dx = \frac{1}{2}\sin 2x$.

$\int_0^{\pi/4} \cos 2x, dx = \left[\frac{1}{2}\sin 2x\right]_0^{\pi/4} = \frac{1}{2}$.

3 $\int \sec^2(4x-1)\,dx = \frac{1}{4}\tan(4x-1) + C$.

4 Use $\cos^2 x = \frac{1+\cos 2x}{2}$.

$\int_0^{\pi/2} \cos^2 x, dx = \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2}$.

The value is $\frac{\pi}{4}$.

5 $\int \frac{5}{2x+1}dx = \frac{5}{2}\ln|2x+1|$.

$\int_1^3 \frac{5}{2x+1}dx = \frac{5}{2}(\ln 7 - \ln 3)$.

The exact value is $\frac{5}{2}\ln\left(\frac{7}{3}\right)$.

6 Here $h = \frac{2-0}{4} = 0.5$.

Trapezium estimate $= \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)$.

$= 0.25(1.000 + 2(0.607) + 2(0.368) + 2(0.223) + 0.135)$.

$= 0.883$ to 3 decimal places.

7 Here $h = 1$.

Estimate $= \frac{1}{2}(0 + 2(0.693) + 2(1.099) + 1.386)$.

$= 2.485$.

8 Area $= \int_0^{\ln 2} e^x, dx$.

$= [e^x]_0^{\ln 2} = 2 - 1 = 1$.

9 $\int 3\sin x, dx = -3\cos x$.

$\int -4\cos 3x, dx = -\dfrac{4}{3}\sin 3x$.

The integral is $-3\cos x - \dfrac{4}{3}\sin 3x + C$.

10 For a concave down curve, the chords lie below the curve.
The trapezium rule therefore gives an under-estimate.