Differentiation

Cambridge International AS & A Level - Mathematics 9709
Paper 2: Pure Mathematics 2 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 2 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Differentiate $y = e^{3x} - 4\ln x + 5\sin 2x$ with respect to x. [4]

2 Differentiate $y = x^2 \ln x$ with respect to x. [4]

3 Find $\frac{dy}{dx}$ when $y = \frac{e^x}{x^2+1}$. [4]

4 The curve $y = xe^{-x}$ has a stationary point. Find its coordinates and determine its nature. [6]

5 A curve is defined parametrically by $x = t^2 - 1$, $y = t^3 + 2t$. Find the gradient of the tangent when $t = 2$. [5]

6 A curve is defined by $x^2 + y^2 = 4xy + 5$. Find $\frac{dy}{dx}$ in terms of x and y. [5]

7 Find the equation of the normal to $y = \ln x$ at the point where $x = e^2$. [5]

8 Differentiate $y = \tan(3x)$ with respect to x. [3]

9 For $y = e^{2x}\cos x$, find $\frac{dy}{dx}$. [4]

10 A curve is defined parametrically by $x = e^t$, $y = t^2$. Find $\frac{dy}{dx}$ in terms of t. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $\dfrac{dy}{dx} = 3e^{3x} - \dfrac{4}{x} + 10\cos 2x$.

2 Use the product rule.

$\dfrac{dy}{dx} = 2x\ln x + x^2 \cdot \dfrac{1}{x}$.

$\dfrac{dy}{dx} = 2x\ln x + x$.

3 Use the quotient rule.

$\dfrac{dy}{dx} = \dfrac{(x^2+1)e^x - e^x(2x)}{(x^2+1)^2}$.

$\dfrac{dy}{dx} = \dfrac{e^x(x^2-2x+1)}{(x^2+1)^2}$.

4 $\dfrac{dy}{dx} = e^{-x} - xe^{-x} = e^{-x}(1-x)$.

At a stationary point, $1 - x = 0$, so $x = 1$.

$y = e^{-1}$.

$\dfrac{dy}{dx}$ changes from positive to negative as x passes through 1.

The stationary point is $(1, e^{-1})$ and it is a maximum.

5 $\dfrac{dx}{dt} = 2t$ and $\dfrac{dy}{dt} = 3t^2 + 2$.

$\dfrac{dy}{dx} = \dfrac{3t^2+2}{2t}$.

At $t = 2$, $\dfrac{dy}{dx} = \dfrac{14}{4} = \dfrac{7}{2}$.

6 Differentiate implicitly: $2x + 2y\dfrac{dy}{dx} = 4y + 4x\dfrac{dy}{dx}$.

$(2y - 4x)\dfrac{dy}{dx} = 4y - 2x$.

$\dfrac{dy}{dx} = \dfrac{4y-2x}{2y-4x} = \dfrac{2y-x}{y-2x}$.

7 At $x = e^2$, $y = 2$.

$\dfrac{dy}{dx} = \dfrac{1}{x}$, so the tangent gradient is e^{-2}.

The normal gradient is $-e^2$.

The normal is $y - 2 = -e^2(x - e^2)$.

8 $\dfrac{d}{dx}\tan(3x) = 3\sec^2(3x)$.

9 Use the product rule.

$$\frac{dy}{dx} = 2e^{2x}\cos x - e^{2x}\sin x \quad .$$

$$\frac{dy}{dx} = e^{2x}(2\cos x - \sin x) \quad .$$

10 $\frac{dx}{dt} = e^t$ and $\frac{dy}{dt} = 2t$.

$$\frac{dy}{dx} = \frac{2t}{e^t} = 2te^{-t} \quad .$$