Trigonometry

Cambridge International AS & A Level - Mathematics 9709
Paper 2: Pure Mathematics 2 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 2 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Given that $\tan A = \frac{3}{4}$ and A is acute, find the exact values of $\sin 2A$ and $\cos 2A$. [5]

2 Show that $(\sec x - \tan x)(\sec x + \tan x) = 1$. [3]

3 Solve $2\sec^2\theta - \tan\theta = 5$ for $0^\circ \le \theta < 360^\circ$. [6]

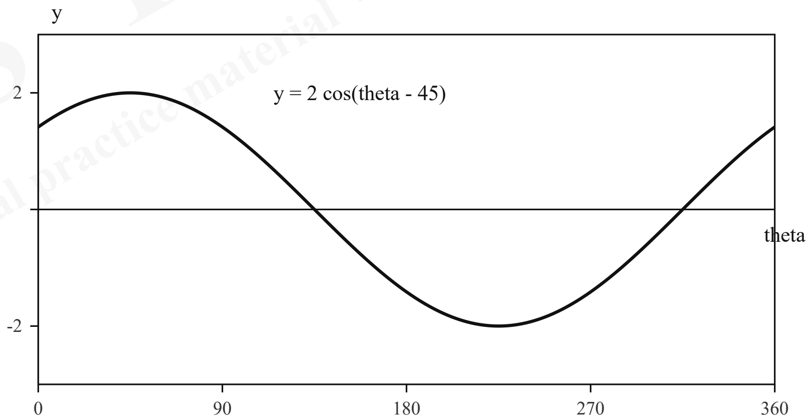
4 Use the addition formula for cosine to find the exact value of $\cos 75^\circ$. [4]

5 Express $5\sin x - 12\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [5]

6 Solve $3\sin x + 4\cos x = 2$ for $0^\circ \le x < 360^\circ$. [6]

7 Solve $\sin 2x = \sin x$ for $0^\circ \le x < 360^\circ$. [5]

8 The graph of $y = 2\cos(\theta - 45^\circ)$ is shown for $0^\circ \le \theta \le 360^\circ$. State its amplitude and the smallest positive value of θ at which the maximum occurs. [4]



Graph for Question 8.

9 Solve $\tan x + \cot x = 3$ for $0^\circ < x < 180^\circ$. [5]

10 Simplify $\cos(x - 30^\circ) - \sqrt{3}\sin(x - 30^\circ)$. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 For an acute angle with $\tan A = \frac{3}{4}$, take $\sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$.

$\sin 2A = 2\sin A\cos A = 2(\frac{3}{5})(\frac{4}{5}) = \frac{24}{25}$.

$\cos 2A = \cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$.

2 $(\sec x - \tan x)(\sec x + \tan x) = \sec^2 x - \tan^2 x$.

Since $\sec^2 x = 1 + \tan^2 x$, this equals 1.

3 Use $\sec^2\theta = 1 + \tan^2\theta$.

$2(1 + \tan^2\theta) - \tan\theta = 5$.

$2\tan^2\theta - \tan\theta - 3 = 0$.

$(2\tan\theta - 3)(\tan\theta + 1) = 0$.

$\tan\theta = \frac{3}{2}$ gives $\theta = 56.3°, 236.3°$.

$\tan\theta = -1$ gives $\theta = 135°, 315°$.

4 $75° = 45° + 30°$.

$\cos 75° = \cos 45°\cos 30° - \sin 45°\sin 30°$.

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$.

$\cos 75° = \frac{\sqrt{6}-\sqrt{2}}{4}$.

5 $R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$.

So $R\cos\alpha = 5$ and $R\sin\alpha = 12$.

$R = 13$ and $\tan\alpha = \frac{12}{5}$.

$\alpha = 67.4°$.

Thus $5\sin x - 12\cos x = 13\sin(x - 67.4°)$.

6 Write $3\sin x + 4\cos x = 5\sin(x+\alpha)$, where $\cos\alpha = \frac{3}{5}$ and $\sin\alpha = \frac{4}{5}$.

Thus $\alpha = 53.1°$.

$5\sin(x + 53.1°) = 2$, so $\sin(x + 53.1°) = 0.4$.

$x + 53.1° = 23.6°$ or $156.4°$ modulo $360°$.

Hence $x = 330.4°$ or $103.3°$.

7 $\sin 2x = 2\sin x\cos x$.

$2\sin x\cos x = \sin x$.

$\sin x(2\cos x - 1) = 0$.

$\sin x = 0$ gives $x = 0°, 180°$.

$\cos x = \frac{1}{2}$ gives $x = 60°, 300°$.

8 The amplitude is 2.

The maximum occurs when $\cos(\theta - 45°) = 1$.

$\theta - 45^\circ = 0^\circ$ modulo 360°.

The smallest positive value is $\theta = 45^\circ$.

9 Let $t = \tan x$.

$t + \dfrac{1}{t} = 3$, so $t^2 - 3t + 1 = 0$.

$t = \dfrac{3 \pm \sqrt{5}}{2}$.

Both roots are positive, so both solutions lie in the first quadrant.

$x = 31.7^\circ$ or 58.3° to 1 decimal place.

10 Use $\cos(x - 30^\circ) = \cos x \cos 30^\circ + \sin x \sin 30^\circ$.

Use $\sin(x - 30^\circ) = \sin x \cos 30^\circ - \cos x \sin 30^\circ$.

Substitution gives $\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) - \sqrt{3}\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right)$.

$= \sqrt{3}\cos x - \sin x$.