Logarithmic and Exponential Functions

Cambridge International AS & A Level - Mathematics 9709
Paper 2: Pure Mathematics 2 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 2 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

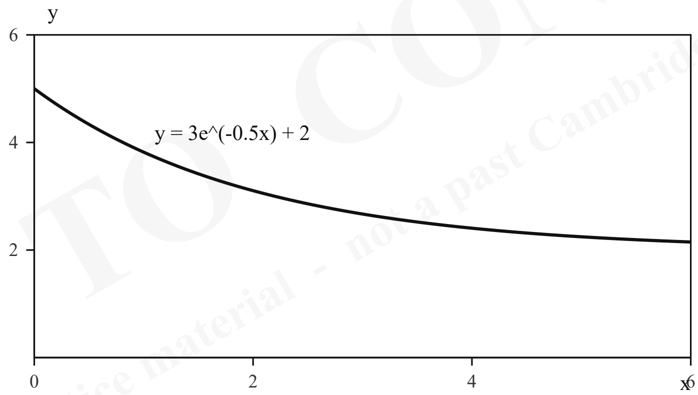
1 Solve $\ln(x-1)+\ln(x+2)=\ln 12$. [5]

2 Solve $e^{2x}-7e^{x}+10=0$. [4]

3 Solve $2^{3x-1}=7$, giving your answer in exact logarithmic form. [4]

4 Solve the inequality $5^{x+2}<40$. [4]

5 The curve $y=3e^{-0.5x}+2$ has a horizontal asymptote. State this asymptote and find the value of x when $y=5$. [5]



Graph for Question 5.

6 Variables x and y are related by $y=kx^{n}$. The points $(2,20)$ and $(8,160)$ lie on the graph. Find k and n. [6]

7 A relationship has the form $y=ka^{x}$. It passes through $(1,12)$ and $(3,108)$. Find k and a. [5]

8 Solve $\ln(2x+1)=3$. [3]

9 Solve $\ln x+\ln 3=2\ln(x-1)$. [6]

10 The equation $\ln y=1.8-0.25x$ represents a straight line when $\ln y$ is plotted against x. Express y in the form Ae^{Bx} and find y when $x=4$. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 The domain requires $x > 1$.

$\ln((x-1)(x+2)) = \ln 12$, so $(x-1)(x+2) = 12$.

$x^2 + x - 2 = 12$, hence $x^2 + x - 14 = 0$.

$x = \frac{-1 \pm \sqrt{57}}{2}$.

Only $x = \frac{-1 + \sqrt{57}}{2}$ satisfies $x > 1$.

2 Let $u = e^x$, where $u > 0$.

$u^2 - 7u + 10 = 0$, so $(u-2)(u-5) = 0$.

$e^x = 2$ or $e^x = 5$.

Therefore $x = \ln 2$ or $x = \ln 5$.

3 Taking logarithms, $(3x-1)\ln 2 = \ln 7$.

$3x - 1 = \frac{\ln 7}{\ln 2}$.

$x = \frac{1}{3}\left(1 + \frac{\ln 7}{\ln 2}\right)$.

4 Since $5 > 1$, logarithms preserve the inequality.

$(x+2)\ln 5 < \ln 40$.

$x + 2 < \frac{\ln 40}{\ln 5}$.

$x < \frac{\ln 40}{\ln 5} - 2$.

5 As $x \to \infty$, $3e^{-0.5x} \to 0$, so the horizontal asymptote is $y = 2$.

When $y = 5$, $3e^{-0.5x} + 2 = 5$.

$e^{-0.5x} = 1$.

So $x = 0$.

6 Using the two points, $\frac{160}{20} = \left(\frac{8}{2}\right)^n$.

$8 = 4^n$, so $n = \frac{3}{2}$.

Using $(2, 20)$, $20 = k2^{3/2}$.

$k = \frac{20}{2\sqrt{2}} = 5\sqrt{2}$.

7 $12 = ka$ and $108 = ka^3$.

Dividing gives $9 = a^2$.

Since $a > 0$, $a = 3$.

$k = 4$.

8 $2x + 1 = e^3$.

$x = \dfrac{e^3 - 1}{2}$.

9 The domain requires $x > 1$.

$\ln(3x) = \ln((x-1)^2)$.

So $3x = (x-1)^2$.

$3x = x^2 - 2x + 1$, hence $x^2 - 5x + 1 = 0$.

$x = \dfrac{5 \pm \sqrt{21}}{2}$.

Only $x = \dfrac{5 + \sqrt{21}}{2}$ is greater than 1.

10 $\ln y = 1.8 - 0.25x$ gives $y = e^{1.8} e^{-0.25x}$.

So $A = e^{1.8}$ and $B = -0.25$.

When $x = 4$, $y = e^{1.8-1} = e^{0.8}$.

$y = 2.23$ to 3 significant figures.