# Algebra

Cambridge International AS & A Level - Mathematics 9709
Paper 2: Pure Mathematics 2 - Exam-style question bank

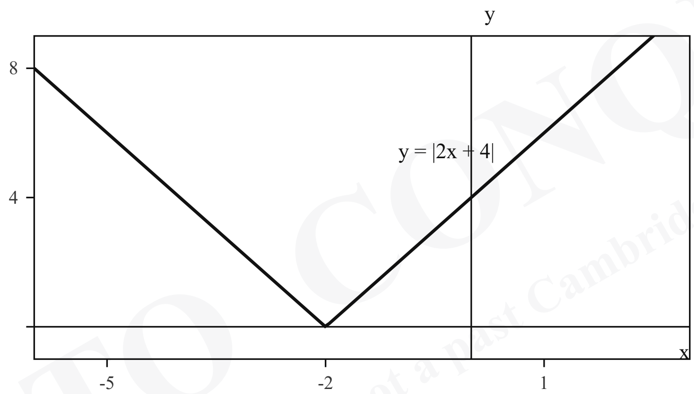
*Original Steps to Conquer practice questions in the style of recent 9709 Paper 2 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.*

## Questions

**1** Solve the equation $|3x - 2| = |x + 4|$. [4]

**2** Solve the inequality $|x - 5| < 2x + 1$. [5]

**3** Sketch the graph of $y = |2x + 4|$, stating the coordinates of its vertex. Hence solve $|2x + 4| = 6$. [5]




*Graph for Question 3.*

**4** Divide $2x^3 - 3x^2 + 5x - 7$ by $x - 2$, giving the quotient and the remainder. [4]

**5** The polynomial $f(x) = x^3 + ax^2 + bx + 6$ has a factor $x - 2$. When divided by $x + 1$, the remainder is $12$. Find $a$ and $b$, and factorise $f(x)$ completely. [8]

**6** The polynomial $P(x) = x^4 - 5x^3 + ax^2 + bx + 12$ has factors $x - 3$ and $x + 1$. Find $a$ and $b$, and hence factorise $P(x)$ completely. [8]

**7** Find the remainder when $3x^3 - 2x^2 + 5x - 4$ is divided by $x^2 + x - 2$. [5]

**8** Divide $x^4 + x^3 - 5x^2 + 2x + 1$ by $x^2 - 2x + 1$. [5]

**9** A polynomial $f(x) = 2x^3 + kx^2 - 11x + 6$ has a factor $2x - 1$. Find $k$. [4]

**10** Find the set of values of $x$ for which $|2x - 1| \geq 5$. [4]

## Worked solutions

*Full methods with final answers. Other valid methods may also earn full marks.*

**1** Squaring both sides gives $(3x-2)^2 = (x+4)^2$.

So $3x-2 = x+4$ or $3x-2 = -(x+4)$.

Hence $x = 3$ or $x = -\frac{1}{2}$.

**2** If $x \geq 5$, then $|x-5| = x-5$, so $x-5 < 2x+1$, which gives $x > -6$. This is true for all $x \geq 5$.

If $x < 5$, then $|x-5| = 5-x$, so $5-x < 2x+1$.

This gives $4 < 3x$, so $x > \frac{4}{3}$.

Combining the cases gives $x > \frac{4}{3}$.

**3** The graph is V-shaped with vertex where $2x+4=0$.

The vertex is $(-2, 0)$.

$|2x+4| = 6$ gives $2x+4 = 6$ or $2x+4=-6$.

Hence $x = 1$ or $x = -5$.

**4** Using polynomial division, $2x^3 - 3x^2 + 5x - 7 = (x-2)(2x^2+x+7)+7$.

The quotient is $2x^2+x+7$ and the remainder is $7$.

**5** $f(2) = 0$ gives $8 + 4a + 2b + 6 = 0$, so $2a + b = -7$.

$f(-1) = 12$ gives $-1 + a - b + 6 = 12$, so $a - b = 7$.

Solving gives $a = 0$ and $b = -7$.

Thus $f(x) = x^3 - 7x + 6$.

Since $x = 1$ is a root, $f(x) = (x-1)(x^2+x-6)$.

Therefore $f(x) = (x-1)(x-2)(x+3)$.

**6** $P(3) = 0$ gives $81 - 135 + 9a + 3b + 12 = 0$, so $3a + b = 14$.

$P(-1) = 0$ gives $1 + 5 + a - b + 12 = 0$, so $a - b = -18$.

Solving gives $a = -1$ and $b = 17$.

$P(x) = x^4 - 5x^3 - x^2 + 17x + 12$.

Dividing by $(x-3)(x+1) = x^2 - 2x - 3$ gives quotient $x^2 - 3x - 4$.

So $P(x) = (x-3)(x+1)(x-4)(x+1)$.

$P(x) = (x-3)(x-4)(x+1)^2$.

**7** $x^2 + x - 2 = (x-1)(x+2)$, so the remainder has form $Ax + B$.

Let $P(x) = 3x^3 - 2x^2 + 5x - 4$.

$P(1) = 2$, so $A + B = 2$.

$P(-2) = -46$, so $-2A + B = -46$.

Solving gives $A = 16$ and $B = -14$.

The remainder is $16x - 14$.

**8** Dividing gives quotient $x^2 + 3x$.

The remainder is $-x + 1$.

Thus $x^4 + x^3 - 5x^2 + 2x + 1 = (x^2 - 2x + 1)(x^2 + 3x) - x + 1$ .

**9** If $2x - 1$ is a factor, then $x = \frac{1}{2}$ is a root.

$f(\frac{1}{2}) = 2(\frac{1}{8}) + k(\frac{1}{4}) - 11(\frac{1}{2}) + 6 = 0$ .

$\frac{1}{4} + \frac{k}{4} - \frac{11}{2} + 6 = 0$ .

$\frac{k}{4} + \frac{3}{4} = 0$ , so $k = -3$ .

**10** $|2x - 1| \geq 5$ gives $2x - 1 \geq 5$ or $2x - 1 \leq -5$ .

So $x \geq 3$ or $x \leq -2$ .

The solution set is $x \leq -2$ or $x \geq 3$ .