Integration

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Find $\int (6x^2 - 4x + 5)\,dx$. [3]

2 Evaluate $\int_1^3 (2x + 1)\,dx$. [3]

3 A curve passes through the point $(1, 4)$ and is such that $\frac{dy}{dx} = 3x^2 - 2$. Find the equation of the curve. [4]

4 Find the area of the region bounded by the curve $y = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 2$. [4]

5 Find $\int (2x - 3)^4\,dx$. [3]

6 Evaluate $\int_1^4 6\sqrt{x}\,dx$. [4]

7 Find the area of the region enclosed between the curve $y = x^2$ and the line $y = x + 2$. [6]

8 The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. [4]

9 Find $\int (x + 1)(x - 3)\,dx$. [3]

10 Evaluate $\int_0^1 (3x^2 - 4x + 2)\,dx$. [3]

11 The region bounded by the line $y = 2x + 1$, the x-axis and the lines $x = 0$ and $x = 2$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed. [5]

12 Find the area of the region enclosed between the curve $y = 8x - x^2$ and the x-axis. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $\int (6x^2 - 4x + 5)\,dx = 6 \cdot \frac{x^3}{3} - 4 \cdot \frac{x^2}{2} + 5x + c$.

$= 2x^3 - 2x^2 + 5x + c$.

2 $\int (2x+1)\,dx = x^2 + x$, evaluated from 1 to 3.

$= (9+3) - (1+1) = 12 - 2 = 10$.

3 $y = \int (3x^2 - 2)\,dx = x^3 - 2x + c$.

At $(1,4)$: $4 = 1 - 2 + c \Rightarrow c = 5$.

$y = x^3 - 2x + 5$.

4 Area $= \int_0^2 (x^2+1)\,dx = \left[\frac{x^3}{3} + x\right]_0^2$.

$= \left(\frac{8}{3} + 2\right) - 0 = \frac{14}{3} \approx 4.67$.

5 $\int (2x-3)^4\,dx = \dfrac{(2x-3)^5}{5 \times 2} + c$.

$= \dfrac{(2x-3)^5}{10} + c$.

6 $6\sqrt{x} = 6x^{1/2}$, so $\int 6x^{1/2}\,dx = 6 \cdot \frac{2}{3}x^{3/2} = 4x^{3/2}$.

From 1 to 4: $4(4^{3/2}) - 4(1) = 4(8) - 4 = 32 - 4 = 28$.

7 Intersections: $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1,\ 2$.

Area $= \int_{-1}^{2} \left((x+2) - x^2\right)dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$.

At $x = 2$: $\frac{10}{3}$. At $x = -1$: $-\frac{7}{6}$.

Area $= \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2} = 4.5$.

8 $V = \pi \int_0^4 y^2\,dx = \pi \int_0^4 x\,dx$.

$= \pi\left[\frac{x^2}{2}\right]_0^4 = \pi(8) = 8\pi$.

9 Expand: $(x+1)(x-3) = x^2 - 2x - 3$.

$\int (x^2 - 2x - 3)\,dx = \frac{x^3}{3} - x^2 - 3x + c$.

10 $\int (3x^2 - 4x + 2)\,dx = x^3 - 2x^2 + 2x$, from 0 to 1.

$= (1 - 2 + 2) - 0 = 1$.

11 $V = \pi \int_0^2 (2x+1)^2 \, dx = \pi \int_0^2 (4x^2 + 4x + 1) \, dx$.

$= \pi \Big[\frac{4x^3}{3} + 2x^2 + x \Big]_0^2 = \pi \left(\frac{32}{3} + 8 + 2 \right) = \pi \left(\frac{32}{3} + 10 \right)$.

$= \frac{62\pi}{3}$.

12 Curve meets x-axis where $8x - x^2 = 0 \Rightarrow x(8-x) = 0 \Rightarrow x = 0,\ 8$.

Area $= \int_0^8 (8x - x^2) \, dx = \Big[4x^2 - \frac{x^3}{3} \Big]_0^8$.

$= 4(64) - \frac{512}{3} = 256 - \frac{512}{3} = \frac{256}{3} \approx 85.3$.