Differentiation

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 The curve has equation $y = 2x^3 - 5x^2 + 4$. Find $\frac{dy}{dx}$ and hence the gradient of the curve at the point where $x = 2$. [3]

2 The curve has equation $y = x^3 - 3x^2 - 9x + 5$. Find the coordinates of the two stationary points and determine the nature of each. [6]

3 Find the equation of the tangent to the curve $y = x^2 - 4x + 1$ at the point where $x = 3$. [4]

4 The curve has equation $y = 2\sqrt{x}$. Find the equation of the normal to the curve at the point where $x = 4$. [4]

5 Find $\frac{dy}{dx}$ for the curve $y = (3x - 1)^5$. [2]

6 The curve has equation $y = x + \frac{4}{x}$. Find the coordinates of the stationary points and determine the nature of each. [6]

7 A cube has side length x cm and volume V cm^3. The side length is increasing at a constant rate of 0.2 cm s^{-1}. Find the rate at which the volume is increasing at the instant when $x = 5$. [3]

8 Find the set of values of x for which the curve $y = x^2 - 6x$ is increasing. [3]

9 Find the set of values of x for which the curve $y = 2x^3 - 9x^2 + 12x$ is decreasing. [4]

10 The curve has equation $y = \frac{18}{2x - 1}$. Find the equation of the tangent to the curve at the point where $x = 2$. [5]

11 The radius r cm of a circle is increasing at a constant rate of 0.3 cm s^{-1}. Find the rate at which the area A cm^2 is increasing at the instant when $r = 10$. Give your answer in terms of π and as a decimal to 3 significant figures. [4]

12 The curve $y = x^3 + ax + b$ has a stationary point at $(2, -3)$, where a and b are constants. Find the values of a and b. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $\frac{dy}{dx} = 6x^2 - 10x$.

At $x = 2$: $6(4) - 10(2) = 24 - 20 = 4$.

2 $\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x-3)(x+1) = 0 \Rightarrow x = 3$ or $x = -1$.

$y(3) = -22$; $y(-1) = 10$.

$\frac{d^2y}{dx^2} = 6x - 6$. At $x = 3$: $12 > 0$ (minimum); at $x = -1$: $-12 < 0$ (maximum).

Minimum $(3, -22)$, maximum $(-1, 10)$.

3 $y(3) = 9 - 12 + 1 = -2$, so the point is $(3, -2)$.

$\frac{dy}{dx} = 2x - 4 = 2$ at $x = 3$.

$y - (-2) = 2(x - 3) \Rightarrow y = 2x - 8$.

4 At $x = 4$: $y = 2\sqrt{4} = 4$. Write $y = 2x^{1/2}$, so $\frac{dy}{dx} = x^{-1/2} = \frac{1}{\sqrt{x}} = \frac{1}{2}$ at $x = 4$.

Normal gradient $= -2$; through $(4, 4)$: $y - 4 = -2(x - 4)$.

$y = -2x + 12$.

5 Chain rule: $\frac{dy}{dx} = 5(3x-1)^4 \times 3$.

$= 15(3x-1)^4$.

6 $y = x + 4x^{-1} \Rightarrow \frac{dy}{dx} = 1 - 4x^{-2} = 1 - \frac{4}{x^2} = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$.

$y(2) = 4$; $y(-2) = -4$.

$\frac{d^2y}{dx^2} = \frac{8}{x^3}$. At $x = 2$: > 0 (minimum); at $x = -2$: < 0 (maximum).

Minimum $(2, 4)$, maximum $(-2, -4)$.

7 $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2 = 75$ at $x = 5$.

$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt} = 75 \times 0.2 = 15$ $\text{cm}^3\ \text{s}^{-1}$.

8 $\frac{dy}{dx} = 2x - 6$.

Increasing $\Rightarrow \frac{dy}{dx} > 0 \Rightarrow 2x - 6 > 0$.

$x > 3$.

9 $\frac{dy}{dx} = 6x^2 - 18x + 12 = 6(x-1)(x-2)$.

Decreasing $\Rightarrow \frac{dy}{dx} < 0 \Rightarrow (x-1)(x-2) < 0$.

$1 < x < 2$.

10 At $x = 2$: $y = \frac{18}{3} = 6$. Write $y = 18(2x-1)^{-1}$.

$\frac{dy}{dx} = 18 \times (-1)(2x-1)^{-2} \times 2 = \frac{-36}{(2x-1)^2} = -4$ at $x = 2$.

$y - 6 = -4(x-2) \Rightarrow y = -4x + 14$.

11 $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r = 20\pi$ at $r = 10$.

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = 20\pi \times 0.3 = 6\pi$.

≈ 18.8 $\text{cm}^2\ \text{s}^{-1}$.

12 $\frac{dy}{dx} = 3x^2 + a = 0$ at $x = 2 \Rightarrow 12 + a = 0 \Rightarrow a = -12$.

The point lies on the curve: $2^3 + 2a + b = -3 \Rightarrow 8 - 24 + b = -3$.

$a = -12$, $b = 13$.