Series

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Find the first three terms, in ascending powers of x, in the expansion of $(2+3x)^5$. [4]

2 Find the coefficient of x^3 in the expansion of $(3-2x)^6$. [3]

3 The coefficient of x^2 in the expansion of $(1+ax)^7$ is 84. Find the two possible values of the constant a. [3]

4 Find the coefficient of x^2 in the expansion of $\left(2+\frac{1}{2}x\right)^8$. [4]

5 The first three terms in the expansion of $(1+px)^n$, in ascending powers of x, are $1+12x+64x^2$. Find the values of the constants p and n. [5]

6 An arithmetic progression has first term 5 and common difference 3. Find (a) the 20th term, [1] (b) the sum of the first 20 terms. [2]

7 The 3rd term of an arithmetic progression is 11 and the 7th term is 27. Find the first term and common difference, and hence the sum of the first 15 terms. [4]

8 A geometric progression has first term 12 and common ratio $\frac{1}{2}$. Find (a) the 5th term, [1] (b) the sum to infinity. [2]

9 The 2nd term of a geometric progression is 6 and the 5th term is 48. Find the common ratio and the first term, and hence the sum of the first 8 terms. [4]

10 The sum to infinity of a geometric progression is 50 and its first term is 10. Find the common ratio and the third term. [4]

11 The sum of the first n terms of an arithmetic progression is $S_n = 2n^2+3n$. Find the first term and the common difference. [4]

12 A geometric progression has first term a and common ratio r, with $|r|<1$. The sum of the first two terms is 9 and the sum to infinity is 12. Find the two possible pairs of values of a and r. [5]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $(2+3x)^5 = 2^5 + \binom{5}{1}2^4(3x) + \binom{5}{2}2^3(3x)^2 + \cdots$

$= 32 + 5(16)(3x) + 10(8)(9x^2) + \cdots$

$= 32 + 240x + 720x^2$.

2 Term in x^3: $\binom{6}{3}(3)^3(-2x)^3 = 20 \times 27 \times (-8)\,x^3$.

Coefficient $= -4320$.

3 Coefficient of x^2 is $\binom{7}{2}a^2 = 21a^2$.

$21a^2 = 84 \Rightarrow a^2 = 4$.

$a = 2$ or $a = -2$.

4 Term in x^2: $\binom{8}{2}(2)^6\left(\frac{1}{2}x\right)^2 = 28 \times 64 \times \frac{1}{4}\,x^2$.

$= 28 \times 16\,x^2$.

Coefficient $= 448$.

5 Coefficient of x: $np = 12$. …(1)

Coefficient of x^2: $\frac{1}{2}n(n-1)p^2 = 64$. …(2)

From (1), $p = \dfrac{12}{n}$. Sub into (2): $\frac{1}{2}n(n-1)\dfrac{144}{n^2} = 64 \Rightarrow \dfrac{72(n-1)}{n} = 64$.

$72(n-1) = 64n \Rightarrow 8n = 72 \Rightarrow n = 9$, then $p = \frac{12}{9} = \frac{4}{3}$.

6 (a) $a_{20} = 5 + 19(3) = 62$.

(b) $S_{20} = \frac{1}{2}(20)(5+62) = 10 \times 67 = 670$.

7 $a + 2d = 11$ and $a + 6d = 27 \Rightarrow 4d = 16 \Rightarrow d = 4$, $a = 3$.

$S_{15} = \frac{1}{2}(15)(2(3) + 14(4)) = \frac{1}{2}(15)(62) = 465$.

8 (a) $a_5 = 12 \times \left(\frac{1}{2}\right)^4 = \dfrac{12}{16} = \frac{3}{4}$.

(b) $S_\infty = \dfrac{12}{1-\frac{1}{2}} = 24$.

9 $ar = 6$ and $ar^4 = 48 \Rightarrow r^3 = 8 \Rightarrow r = 2$, then $a = 3$.

$S_8 = \dfrac{3(2^8-1)}{2-1} = 3(255) = 765$.

10 $S_\infty = \dfrac{a}{1-r}$: $\dfrac{10}{1-r} = 50 \Rightarrow 1 - r = \frac{1}{5} \Rightarrow r = \frac{4}{5}$.

$a_3 = 10 \times \left(\frac{4}{5}\right)^2 = 10 \times \frac{16}{25} = 6.4$.

11 $S_1 = 2 + 3 = 5$, so the first term $a = 5$.

$S_2 = 8 + 6 = 14$, so the second term $= S_2 - S_1 = 9$, giving $d = 9 - 5 = 4$.

12 $a(1+r)=9$ $\quad\ldots(1)$ and $\dfrac{a}{1-r}=12 \Rightarrow a=12(1-r)$ $\quad\ldots(2)$.

Sub (2) into (1): $12(1-r)(1+r)=9 \Rightarrow 12(1-r^2)=9 \Rightarrow r^2=\frac{1}{4} \Rightarrow r=\pm\frac{1}{2}$.

$r=\frac{1}{2} \Rightarrow a=6$; $r=-\frac{1}{2} \Rightarrow a=18$.

$(a,r)=\left(6,\frac{1}{2}\right)$ or $\left(18,-\frac{1}{2}\right)$.