Trigonometry

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Solve the equation $3\sin x = 2\cos x$ for $0^\circ \le x \le 360^\circ$. [3]

2 Solve the equation $2\cos^2 x - \cos x - 1 = 0$ for $0^\circ \le x \le 360^\circ$. [4]

3 Prove the identity $\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$. [3]

4 Solve the equation $2\sin^2 x + 3\cos x = 0$ for $0^\circ \le x \le 360^\circ$. [4]

5 Prove the identity $\frac{3}{1+\sin\theta} + \frac{3}{1-\sin\theta} \equiv \frac{6}{\cos^2\theta}$. [3]

6 Solve the equation $\cos(x - 40^\circ) = 0.5$ for $0^\circ \le x \le 360^\circ$. [4]

7 Solve the equation $5\sin^2\theta + 2\cos\theta = 5$ for $0^\circ \le \theta \le 360^\circ$. [5]

8 The function f is defined by $f(x) = 3 + 2\sin x$ for $0^\circ \le x \le 360^\circ$. (a) Write down the greatest and least values of $f(x)$. [2] (b) State the number of solutions of $f(x) = 4$ in this interval. [1]

9 Solve the equation $6\sin^2 x - \sin x - 1 = 0$ for $0^\circ \le x \le 360^\circ$. [5]

10 Solve the equation $2\cos^2 x = \sin x + 1$ for $0^\circ \le x \le 360^\circ$. [5]

11 Solve the equation $\tan x = \sqrt{3}$ for $0 \le x \le 2\pi$, giving your answers in radians as exact multiples of π. [3]

12 Solve the equation $4\sin x = 3\cos x$ for $-180^\circ \le x \le 180^\circ$. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 Divide by $\cos x$: $3\tan x = 2 \Rightarrow \tan x = \frac{2}{3}$.

Basic angle $= 33.69^\circ$; $\tan$ is positive in the 1st and 3rd quadrants.

$x = 33.7^\circ$ or 213.7°.

2 Factorise: $(2\cos x + 1)(\cos x - 1) = 0 \Rightarrow \cos x = -\frac{1}{2}$ or $\cos x = 1$.

$\cos x = -\frac{1}{2} \Rightarrow x = 120^\circ$ or 240°.

$\cos x = 1 \Rightarrow x = 0^\circ$ or 360°.

$x = 0^\circ,\ 120^\circ,\ 240^\circ,\ 360^\circ$.

3 LHS $= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$.

Since $\sin^2\theta + \cos^2\theta = 1$, this equals $\dfrac{1}{\sin\theta\cos\theta} =$ RHS.

4 Use $\sin^2 x = 1 - \cos^2 x$: $2(1 - \cos^2 x) + 3\cos x = 0 \Rightarrow 2\cos^2 x - 3\cos x - 2 = 0$.

$(2\cos x + 1)(\cos x - 2) = 0 \Rightarrow \cos x = -\frac{1}{2}$ (no solution from $\cos x = 2$).

$x = 120^\circ$ or 240°.

5 LHS $= \dfrac{3(1 - \sin\theta) + 3(1 + \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)} = \dfrac{6}{1 - \sin^2\theta}$.

Since $1 - \sin^2\theta = \cos^2\theta$, LHS $= \dfrac{6}{\cos^2\theta} =$ RHS.

6 Let $u = x - 40^\circ$, so $-40^\circ \le u \le 320^\circ$ and $\cos u = 0.5$.

Basic angle 60°: $u = 60^\circ$ or $u = -60^\circ$ (i.e. 300°, within range).

$x = u + 40^\circ \Rightarrow x = 100^\circ$ or $x = 340^\circ$.

7 $5(1 - \cos^2\theta) + 2\cos\theta - 5 = 0 \Rightarrow \cos\theta(2 - 5\cos\theta) = 0$.

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$ or 270°.

$\cos\theta = \frac{2}{5} = 0.4 \Rightarrow$ basic angle 66.4°, so $\theta = 66.4^\circ$ or 293.6°.

$\theta = 66.4^\circ,\ 90^\circ,\ 270^\circ,\ 293.6^\circ$.

8 (a) $\sin x$ ranges from -1 to 1, so f ranges from 1 to 5: greatest 5, least 1.

(b) $f(x) = 4 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = 30^\circ$ or 150°: 2 solutions.

9 Factorise: $(3\sin x + 1)(2\sin x - 1) = 0 \Rightarrow \sin x = -\frac{1}{3}$ or $\sin x = \frac{1}{2}$.

$\sin x = \frac{1}{2} \Rightarrow x = 30^\circ$ or 150°.

$\sin x = -\frac{1}{3} \Rightarrow$ basic angle 19.47°, $\sin$ negative in 3rd/4th quadrants: $x = 199.5^\circ$ or 340.5°.

$x = 30^\circ,\ 150^\circ,\ 199.5^\circ,\ 340.5^\circ$.

10 $2(1 - \sin^2 x) = \sin x + 1 \Rightarrow 2\sin^2 x + \sin x - 1 = 0$.

$(2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2}$ or $\sin x = -1$.

$\sin x = \frac{1}{2} \Rightarrow x = 30^\circ$ or 150°; $\sin x = -1 \Rightarrow x = 270^\circ$.

$x = 30^\circ,\ 150^\circ,\ 270^\circ$.

11 $\tan x = \sqrt{3} \Rightarrow$ basic angle $\frac{\pi}{3}$; $\tan$ positive in the 1st and 3rd quadrants.
$x = \frac{\pi}{3}$ or $\frac{4\pi}{3}$.

12 $\tan x = \frac{3}{4} \Rightarrow$ basic angle 36.87° .
Within $-180^\circ \leq x \leq 180^\circ$: 1st quadrant 36.9° and 3rd quadrant $36.87^\circ - 180^\circ = -143.1^\circ$.
$x = 36.9^\circ$ or -143.1° .