

Circular Measure

Cambridge International AS & A Level - Mathematics 9709

Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets []. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

- 1 A sector of a circle has radius 8 cm and angle 0.9 radians at the centre. Find (a) the length of the arc, [2] (b) the area of the sector. [2]
- 2 The arc of a sector of a circle has length 15 cm and the area of the sector is 67.5 cm^2 . Find the radius of the circle and the angle of the sector in radians. [4]
- 3 A chord AB of a circle of radius 10 cm and centre O subtends an angle of 1.2 radians at O . Find (a) the area of the minor sector OAB , [2] (b) the area of triangle OAB , [2] (c) the area of the minor segment, to 3 significant figures. [1]
- 4 (a) Express 144° in radians, giving your answer as an exact multiple of π . [1] (b) Express $\frac{5\pi}{6}$ radians in degrees. [1]
- 5 A sector of a circle has perimeter 28 cm and radius 8 cm. Find the angle of the sector in radians and the area of the sector. [4]
- 6 Two radii OA and OB of a circle of radius 6 cm enclose an angle $AOB = \frac{2\pi}{3}$ radians. Find the exact perimeter of the minor segment cut off by the chord AB . [4]
- 7 A sector of a circle has area 30 cm^2 and arc length 12 cm. Find the radius of the circle and the angle of the sector in radians. [4]
- 8 An annular sector lies between an inner radius of 4 cm and an outer radius of 9 cm, with an angle of 0.8 radians at the common centre. Find (a) its area, [3] (b) its perimeter. [3]
- 9 A sector of a circle has radius 7 cm and angle 40° at the centre. Find the arc length and the area of the sector, each to 3 significant figures. [4]
- 10 The minor arc of a circle of radius 15 cm has length 6π cm. Find (a) the angle subtended at the centre, as an exact multiple of π radians, [2] (b) the exact area of the corresponding minor sector. [2]
- 11 A chord PQ of length 9 cm is drawn in a circle of radius 9 cm and centre O . Find the area of the minor segment cut off by PQ , to 3 significant figures. [5]
- 12 A sector of a circle has radius 6 cm and angle θ radians. Given that the numerical value of the

area of the sector equals the numerical value of its perimeter, find θ . [4]

STEPS TO CONQUER
original practice material - not a past Cambridge paper

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

- 1 (a) Arc = $r\theta = 8 \times 0.9 = 7.2$ cm.
 (b) Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times 0.9 = 28.8$ cm².

- 2 $r\theta = 15$ and $\frac{1}{2}r^2\theta = 67.5$.
 Dividing: $\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{67.5}{15} \Rightarrow \frac{r}{2} = 4.5 \Rightarrow r = 9$ cm.
 $\theta = \frac{15}{9} = \frac{5}{3} \approx 1.67$ rad.

- 3 (a) Sector = $\frac{1}{2} \times 10^2 \times 1.2 = 60$ cm².
 (b) Triangle = $\frac{1}{2} \times 10 \times 10 \times \sin 1.2 = 46.6$ cm² (3 s.f.).
 (c) Segment = sector – triangle = $60 - 46.60 = 13.4$ cm².

- 4 (a) $144 \times \frac{\pi}{180} = \frac{4\pi}{5}$.
 (b) $\frac{5\pi}{6} \times \frac{180}{\pi} = 150^\circ$.

- 5 Perimeter = $2r + r\theta = 16 + 8\theta = 28 \Rightarrow 8\theta = 12 \Rightarrow \theta = 1.5$ rad.
 Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 64 \times 1.5 = 48$ cm².

- 6 Arc AB = $r\theta = 6 \times \frac{2\pi}{3} = 4\pi$ cm.
 Chord AB = $2r \sin\left(\frac{\theta}{2}\right) = 2 \times 6 \times \sin \frac{\pi}{3} = 6\sqrt{3}$ cm.
 Perimeter = $(4\pi + 6\sqrt{3})$ cm ≈ 23.0 cm.

- 7 $\frac{1}{2}r^2\theta = 30$ and $r\theta = 12$.
 $\frac{1}{2}r^2\theta = \frac{1}{2}r(r\theta) = \frac{1}{2}r(12) = 6r = 30 \Rightarrow r = 5$ cm.
 $\theta = \frac{12}{5} = 2.4$ rad.

- 8 (a) Area = $\frac{1}{2}\theta(R^2 - r^2) = \frac{1}{2} \times 0.8 \times (81 - 16) = 26$ cm².
 (b) Perimeter = outer arc + inner arc + 2 $\times$ straight edge
 = $0.8 \times 9 + 0.8 \times 4 + 2 \times (9 - 4) = 7.2 + 3.2 + 10 = 20.4$ cm.

- 9 $40^\circ = 40 \times \frac{\pi}{180} = \frac{2\pi}{9} \approx 0.6981$ rad.
 Arc = $7 \times 0.6981 = 4.89$ cm.
 Area = $\frac{1}{2} \times 7^2 \times 0.6981 = 17.1$ cm².

- 10 (a) $\theta = \frac{\text{arc}}{r} = \frac{6\pi}{15} = \frac{2\pi}{5}$ rad.
 (b) Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 225 \times \frac{2\pi}{5} = 45\pi$ cm² ≈ 141 cm².

- 11 Chord = $2r \sin\left(\frac{\theta}{2}\right)$: $9 = 18 \sin\left(\frac{\theta}{2}\right) \Rightarrow \sin\left(\frac{\theta}{2}\right) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$.

$$\begin{aligned}\text{Segment} &= \frac{1}{2}r^2(\theta - \sin \theta) = \frac{1}{2} \times 81 \times \left(\frac{\pi}{3} - \sin \frac{\pi}{3}\right) \\ &= 40.5 \times (1.04720 - 0.86603) = 7.34 \quad \text{cm}^2.\end{aligned}$$

12 $\text{Area} = \frac{1}{2} \times 6^2 \times \theta = 18\theta$; Perimeter = $12 + 6\theta$.

$$18\theta = 12 + 6\theta \Rightarrow 12\theta = 12$$

$$\theta = 1 \text{ rad.}$$