Coordinate Geometry

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 The points $A(-1, 4)$ and $B(5, -2)$ are given. (a) Find the midpoint of AB. [1] (b) Find the exact length of AB. [2] (c) Find the equation of the perpendicular bisector of AB, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [3]

2 The line L passes through the point $(3, -2)$ and is perpendicular to the line $2x + 5y = 10$. Find the equation of L, giving your answer in the form $y = mx + c$. [4]

3 The triangle ABC has vertices $A(0, 1)$, $B(6, 3)$ and $C(4, k)$. Given that the triangle is right-angled at B, find the value of k. [4]

4 A circle has equation $(x - 2)^2 + (y + 3)^2 = 25$. (a) Write down the centre and radius. [2] (b) Show that the point $P(5, 1)$ lies on the circle. [1] (c) Find the equation of the tangent to the circle at P, in the form $ax + by + c = 0$. [3]

5 A circle has centre $(4, 1)$ and passes through the point $(7, 5)$. Find the equation of the circle. [3]

6 Find the centre and radius of the circle with equation $x^2 + y^2 - 6x + 8y - 11 = 0$. [3]

7 The points $P(2, 7)$ and $Q(8, -1)$ are the ends of a diameter of a circle. Find the equation of the circle. [4]

8 The line $y = x + 2$ intersects the circle $x^2 + y^2 = 10$ at two points. Find the coordinates of these two points. [4]

9 The line $3x + 4y = 25$ is a tangent to the circle $x^2 + y^2 = r^2$, which has centre the origin O. Find r. [3]

10 The points $A(2, -1)$ and $B(8, 7)$ both lie on a circle whose centre lies on the line $x = 5$. Find the equation of the circle. [5]

11 Find the two values of the constant k for which the line $y = kx + 4$ is a tangent to the circle $x^2 + y^2 = 4$. [4]

12 A circle has centre $C(5, 2)$ and radius 3. Find the exact length of the tangent drawn from the external point $T(10, 14)$ to the circle. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 (a) Midpoint $= \left(\dfrac{-1+5}{2}, \dfrac{4+(-2)}{2}\right) = (2, 1)$.

(b) $AB = \sqrt{(5+1)^2 + (-2-4)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$.

(c) Gradient of $AB = \dfrac{-2-4}{5+1} = -1$, so the bisector has gradient 1, through $(2, 1)$:

$y - 1 = 1(x-2) \Rightarrow x - y - 1 = 0$.

2 $2x + 5y = 10 \Rightarrow y = -\frac{2}{5}x + 2$, gradient $-\frac{2}{5}$.

Perpendicular gradient $= \frac{5}{2}$. Through $(3, -2)$: $y + 2 = \frac{5}{2}(x-3)$.

$y = \frac{5}{2}x - \frac{19}{2}$.

3 $\vec{BA} = A - B = (-6, -2)$; $\vec{BC} = C - B = (-2, k-3)$.

Right angle at $B \Rightarrow \vec{BA} \cdot \vec{BC} = 0$: $(-6)(-2) + (-2)(k-3) = 0$.

$12 - 2(k-3) = 0 \Rightarrow 2k = 18$.

$k = 9$.

4 (a) Centre $(2, -3)$, radius 5.

(b) $(5-2)^2 + (1+3)^2 = 9 + 16 = 25$, so P lies on the circle.

(c) Gradient of radius to $P = \dfrac{1+3}{5-2} = \frac{4}{3}$, so tangent gradient $= -\frac{3}{4}$.

$y - 1 = -\frac{3}{4}(x-5) \Rightarrow 4y - 4 = -3x + 15 \Rightarrow 3x + 4y - 19 = 0$.

5 $r^2 = (7-4)^2 + (5-1)^2 = 9 + 16 = 25$.

$(x-4)^2 + (y-1)^2 = 25$.

6 Complete the square: $(x-3)^2 - 9 + (y+4)^2 - 16 - 11 = 0$.

$(x-3)^2 + (y+4)^2 = 36$.

Centre $(3, -4)$, radius 6.

7 Centre = midpoint of $PQ = (5, 3)$.

$PQ = \sqrt{(8-2)^2 + (-1-7)^2} = \sqrt{36+64} = 10$, so radius $= 5$.

$(x-5)^2 + (y-3)^2 = 25$.

8 Substitute: $x^2 + (x+2)^2 = 10 \Rightarrow 2x^2 + 4x + 4 = 10 \Rightarrow x^2 + 2x - 3 = 0$.

$(x+3)(x-1) = 0 \Rightarrow x = -3$ or $x = 1$; then $y = x + 2$.

$(-3, -1)$ and $(1, 3)$.

9 The perpendicular from O to the line is in the direction $(3, 4)$: points $(3t, 4t)$.

On the line: $3(3t) + 4(4t) = 25 \Rightarrow 25t = 25 \Rightarrow t = 1$, so the point of contact is $(3, 4)$.

$r = \sqrt{3^2 + 4^2} = 5$.

10 Let the centre be $(5, k)$. Equal distances to A and B:

$(5-2)^2+(k+1)^2=(5-8)^2+(k-7)^2 \Rightarrow 9+(k+1)^2=9+(k-7)^2$.

$(k+1)^2=(k-7)^2 \Rightarrow 2k+1=-14k+49 \Rightarrow 16k=48 \Rightarrow k=3$.

Centre $(5,3)$; $r^2=(5-2)^2+(3+1)^2=25$, so $(x-5)^2+(y-3)^2=25$.

11 Substitute: $x^2+(kx+4)^2=4 \Rightarrow (1+k^2)x^2+8kx+12=0$.

Tangent $\Rightarrow$ discriminant $=0$: $(8k)^2-4(1+k^2)(12)=0$.

$64k^2-48-48k^2=0 \Rightarrow 16k^2=48 \Rightarrow k^2=3$.

$k=\pm\sqrt{3}$.

12 $CT^2=(10-5)^2+(14-2)^2=25+144=169$.

The tangent is perpendicular to the radius at the point of contact, so length $=\sqrt{CT^2-r^2}$.

$=\sqrt{169-9}=\sqrt{160}=4\sqrt{10}$.