Functions

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 The function f is defined by $f(x) = 3x - 5$ for $x \in \mathbb{R}$. Find an expression for $f^{-1}(x)$ and solve the equation $f(x) = f^{-1}(x)$. [4]

2 The function f is defined by $f(x) = x^2 - 4x + 7$ for $x \geq 2$. (a) Express $f(x)$ in the form $(x-a)^2 + b$ and write down the range of f. [3] (b) Explain why f^{-1} exists, and find an expression for $f^{-1}(x)$, stating its domain. [3]

3 The functions f and g are defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 3$ for $x \in \mathbb{R}$. (a) Find and simplify $fg(x)$. [2] (b) Find and simplify $gf(x)$. [2] (c) Solve $fg(x) = 27$. [2]

4 Describe fully a sequence of three transformations that maps the curve $y = x^3$ onto the curve $y = 2(x-1)^3 + 5$, making clear the order in which they are applied. [4]

5 The functions f and g are defined by $f(x) = 5 - 2x$ for $x \in \mathbb{R}$ and $g(x) = \dfrac{x}{x-1}$ for $x \neq 1$. Find an expression for $gf(x)$, and state the value of x that must be excluded from its domain. [4]

6 The function f is defined by $f(x) = x^2 - 6x + 10$ for $x \geq k$. (a) Given that f is one-one, find the least possible value of the constant k. [2] (b) Using this value of k, find an expression for $f^{-1}(x)$ and state its domain. [3]

7 The function f is defined by $f(x) = \dfrac{1}{x-2}$ for $x > 2$. Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

8 The functions f and g are defined for $x \geq 0$ by $f(x) = \sqrt{x}$ and $g(x) = 2x + 3$. Solve the equation $gf(x) = 10$, giving your answer as an exact fraction. [3]

9 The curve $y = f(x)$ passes through the point $(2, 5)$. Write down the coordinates of the corresponding point on each of the curves (a) $y = f(x-3)$, (b) $y = 2f(x)$, (c) $y = f(-x)$. [3]

10 The function f is defined by $f(x) = 4 - (x-1)^2$ for $x \in \mathbb{R}$. (a) State the range of f and explain why f does not have an inverse. [2] (b) The domain is now restricted to $x \geq 1$. Find $f^{-1}(x)$ for this restricted function and state its domain. [3]

11 The function f is defined by $f(x) = 3x - 2$ for $x \in \mathbb{R}$. Find and simplify $ff(x)$, and hence solve $ff(x) = x$. [3]

12 The function f is defined by $f(x) = ax + b$ for $x \in \mathbb{R}$, where a and b are constants. Given that $f(1) = 5$ and $f^{-1}(11) = 3$, find the values of a and b. [4]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $y = 3x - 5 \Rightarrow x = \dfrac{y+5}{3}$, so $f^{-1}(x) = \dfrac{x+5}{3}$.

$3x - 5 = \dfrac{x+5}{3} \Rightarrow 9x - 15 = x + 5 \Rightarrow 8x = 20$.

$x = 2.5$.

2 (a) $x^2 - 4x + 7 = (x-2)^2 + 3$, so $a = 2$, $b = 3$. Range: $f(x) \geq 3$.

(b) For $x \geq 2$ the function is increasing, hence one-one, so f^{-1} exists.

$y = (x-2)^2 + 3 \Rightarrow x = 2 + \sqrt{y-3}$ (positive root, since $x \geq 2$).

$f^{-1}(x) = 2 + \sqrt{x-3}$, with domain $x \geq 3$.

3 (a) $fg(x) = 2(x^2 - 3) + 1 = 2x^2 - 5$.

(b) $gf(x) = (2x+1)^2 - 3 = 4x^2 + 4x - 2$.

(c) $2x^2 - 5 = 27 \Rightarrow x^2 = 16$.

$x = \pm 4$.

4 Start from $y = x^3$.

1. Translate 1 unit in the positive x-direction: $y = (x-1)^3$.
2. Stretch parallel to the y-axis, scale factor 2: $y = 2(x-1)^3$.
3. Translate 5 units in the positive y-direction: $y = 2(x-1)^3 + 5$.

5 $gf(x) = \dfrac{f(x)}{f(x) - 1} = \dfrac{5 - 2x}{(5 - 2x) - 1} = \dfrac{5 - 2x}{4 - 2x}$.

Undefined where the denominator is 0: $4 - 2x = 0 \Rightarrow x = 2$.

$x = 2$ must be excluded.

6 $f(x) = (x-3)^2 + 1$, with vertex at $x = 3$.

(a) f is one-one for $x \geq 3$, so the least value is $k = 3$.

(b) $y = (x-3)^2 + 1 \Rightarrow x = 3 + \sqrt{y-1}$.

$f^{-1}(x) = 3 + \sqrt{x-1}$, with domain $x \geq 1$.

7 $y = \dfrac{1}{x-2} \Rightarrow x - 2 = \dfrac{1}{y} \Rightarrow x = 2 + \dfrac{1}{y}$.

$f^{-1}(x) = 2 + \dfrac{1}{x}$.

For $x > 2$, $f(x)$ takes all positive values, so the domain of f^{-1} is $x > 0$.

8 $gf(x) = 2\sqrt{x} + 3 = 10 \Rightarrow \sqrt{x} = \dfrac{7}{2}$.

$x = \left(\dfrac{7}{2}\right)^2$.

$x = \dfrac{49}{4}$.

9 (a) Translation 3 right: $(5,5)$.

(b) Stretch factor 2 parallel to the y-axis: $(2,10)$.

(c) Reflection in the y-axis: $(-2,5)$.

10 (a) Maximum value 4 at $x=1$, so the range is $f(x) \le 4$. f is many-one (e.g. $f(0)=f(2)=3$), so it has no inverse.

(b) For $x \ge 1$: $y=4-(x-1)^2 \Rightarrow (x-1)^2=4-y \Rightarrow x=1+\sqrt{4-y}$.

$f^{-1}(x)=1+\sqrt{4-x}$, with domain $x \le 4$.

11 $ff(x)=3(3x-2)-2=9x-8$.

$9x-8=x \Rightarrow 8x=8$.

$x=1$.

12 $f(1)=5$: $a+b=5$ . ...(1)

$f^{-1}(11)=3$ means $f(3)=11$: $3a+b=11$ . ...(2)

$(2)-(1)$: $2a=6 \Rightarrow a=3$, then $b=2$.

$a=3$, $b=2$ (so $f(x)=3x+2$).