Quadratics

Cambridge International AS & A Level - Mathematics 9709
Paper 1: Pure Mathematics 1 - Exam-style question bank

Original Steps to Conquer practice questions in the style of recent 9709 Paper 1 examinations - not past Cambridge questions. Marks are shown in brackets [ ]. Give non-exact answers to 3 significant figures (1 d.p. for angles) and show full working.

Questions

1 Express $2x^2 - 12x + 23$ in the form $a(x-b)^2 + c$, where a, b and c are constants. Hence state the least value of $2x^2 - 12x + 23$ and the value of x at which it occurs. [4]

2 The equation $3x^2 + (k+2)x + 12 = 0$ has a repeated root, where k is a constant. Find the two possible values of k. [4]

3 Find the set of values of x for which $2x^2 + 5x - 12 \leq 0$. [4]

4 The line $y = 2x + c$ is a tangent to the curve $y = x^2 - 3x + 7$. Find the value of the constant c. [4]

5 The line $y = mx - 5$ meets the curve $y = x^2 - 3x + 4$ at two distinct points. Find the set of values of the constant m. [5]

6 Solve the equation $x - 7\sqrt{x} + 10 = 0$. [4]

7 Solve the equation $x^4 - 13x^2 + 36 = 0$. [4]

8 The curve $y = x^2 + (k-3)x + (k+1)$ crosses the x-axis at two distinct points, where k is a constant. Find the set of values of k, giving your answer in an exact form. [5]

9 (a) Express $x^2 + 6x + 1$ in the form $(x+p)^2 + q$. [2] (b) Hence solve $x^2 + 6x + 1 = 0$, giving your answers in the form $a \pm b\sqrt{2}$ where a and b are integers. [2]

10 The quadratic expression $5 + 4x - x^2$ is to be written in the form $a - (x-b)^2$. (a) Find the constants a and b. [3] (b) State the greatest value of $5 + 4x - x^2$ and the value of x at which it occurs, and write down the range of values taken by the expression. [3]

11 The curve $y = x^2 + ax + b$ passes through the point $(1, 0)$ and has a least value of -4, where a and b are constants. Find the two possible pairs of values of a and b. [5]

12 The line $y = 2x + 5$ intersects the curve $y = x^2 + 4$ at the points A and B. Find the coordinates of A and B, and show that the length of AB is $2\sqrt{10}$. [6]

Worked solutions

Full methods with final answers. Other valid methods may also earn full marks.

1 $2x^2 - 12x + 23 = 2(x^2 - 6x) + 23 = 2\big((x-3)^2 - 9\big) + 23$

$= 2(x-3)^2 - 18 + 23 = 2(x-3)^2 + 5$, so $a = 2$, $b = 3$, $c = 5$.

Since $2(x-3)^2 \geq 0$, the least value is 5, occurring at $x = 3$.

2 Repeated root $\Rightarrow$ discriminant $= 0$: $(k+2)^2 - 4(3)(12) = 0$.

$(k+2)^2 = 144 \Rightarrow k + 2 = \pm 12$.

$k = 10$ or $k = -14$.

3 Factorise: $2x^2 + 5x - 12 = (2x-3)(x+4)$.

Roots at $x = \frac{3}{2}$ and $x = -4$; the parabola opens upwards, so the expression is ≤ 0 between the roots.

$-4 \leq x \leq \frac{3}{2}$.

4 Set equal: $x^2 - 3x + 7 = 2x + c \Rightarrow x^2 - 5x + (7 - c) = 0$.

Tangent $\Rightarrow$ discriminant $= 0$: $25 - 4(7 - c) = 0$.

$25 - 28 + 4c = 0 \Rightarrow 4c = 3$.

$c = \frac{3}{4}$.

5 $x^2 - 3x + 4 = mx - 5 \Rightarrow x^2 - (3+m)x + 9 = 0$.

Two distinct points $\Rightarrow$ discriminant > 0: $(3+m)^2 - 36 > 0$.

$(m+3)^2 > 36 \Rightarrow m + 3 > 6$ or $m + 3 < -6$.

$m > 3$ or $m < -9$.

6 Let $u = \sqrt{x} \quad (u \geq 0)$, so $u^2 - 7u + 10 = 0$.

$(u-2)(u-5) = 0 \Rightarrow u = 2$ or $u = 5$.

Then $\sqrt{x} = 2$ or $\sqrt{x} = 5$.

$x = 4$ or $x = 25$.

7 Let $u = x^2$, so $u^2 - 13u + 36 = 0$.

$(u-4)(u-9) = 0 \Rightarrow u = 4$ or $u = 9$.

$x^2 = 4$ or $x^2 = 9$.

$x = \pm 2$ or $x = \pm 3$.

8 Two distinct roots $\Rightarrow$ discriminant > 0: $(k-3)^2 - 4(k+1) > 0$.

$k^2 - 6k + 9 - 4k - 4 > 0 \Rightarrow k^2 - 10k + 5 > 0$.

Critical values: $k = \dfrac{10 \pm \sqrt{100 - 20}}{2} = \dfrac{10 \pm 4\sqrt{5}}{2} = 5 \pm 2\sqrt{5}$.

$k < 5 - 2\sqrt{5}$ or $k > 5 + 2\sqrt{5}$.

9 (a) $x^2 + 6x + 1 = (x+3)^2 - 9 + 1 = (x+3)^2 - 8$.

(b) $(x+3)^2 - 8 = 0 \Rightarrow (x+3)^2 = 8 \Rightarrow x + 3 = \pm\sqrt{8} = \pm 2\sqrt{2}$.

$x = -3 \pm 2\sqrt{2}$.

10 (a) $5+4x-x^2=-(x^2-4x)+5=-\big((x-2)^2-4\big)+5=9-(x-2)^2$. So $a=9$, $b=2$.

(b) Greatest value 9, at $x=2$ (since $(x-2)^2\ge 0$).

Range: the expression takes all values ≤ 9.

11 Passes through $(1,0)$: $1+a+b=0$, so $b=-1-a$. …(1)

Least value of x^2+ax+b is $b-\frac{a^2}{4}=-4$. …(2)

Sub (1) into (2): $(-1-a)-\frac{a^2}{4}=-4\Rightarrow a^2+4a-12=0$.

$(a+6)(a-2)=0\Rightarrow a=-6$ or $a=2$; then $b=-1-a$.

$(a,b)=(-6,5)$ or $(a,b)=(2,-3)$.

12 $x^2+4=2x+5\Rightarrow x^2-2x-1=0\Rightarrow x=\dfrac{2\pm\sqrt{8}}{2}=1\pm\sqrt{2}$.

$y=2x+5$: at $x=1+\sqrt{2}$, $y=7+2\sqrt{2}$; at $x=1-\sqrt{2}$, $y=7-2\sqrt{2}$.

$A=(1+\sqrt{2},\ 7+2\sqrt{2})$, $B=(1-\sqrt{2},\ 7-2\sqrt{2})$.

$\Delta x=2\sqrt{2}$, $\Delta y=4\sqrt{2}\Rightarrow AB=\sqrt{(2\sqrt{2})^2+(4\sqrt{2})^2}=\sqrt{8+32}=\sqrt{40}=2\sqrt{10}$.